Calculation Policy
Park Primary School
(2019)

(Aligned with 2014 National Curriculum and Maths-No Problem!)

Introduction:

This policy sets out the methods used to help our pupils, families and staff in understanding and progressing through the strategies taught across the four operations (addition, subtraction, multiplication and division). It has been devised in order to be in line with the National Curriculum (2014) and with the expectation that each child will develop a mastery level of understanding in mathematics. We are facilitating the teaching of mathematics in Key Stage 1 & 2 using the ***Maths – No Problem!*** series.

The national curriculum for mathematics aims to ensure that all pupils:

- Become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **Reason** mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.
- Can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Maths-No Problem!

Maths – No Problem! is a comprehensive series of textbooks that adopts a spiral design with carefully build-up mathematical concepts and processes adapted from the maths mastery approaches used in Singapore. The *Concrete-Pictorial-Abstract* (C-P-A) approach forms an integral part of the learning process through the materials developed for this series.

What is mastery?

A mathematical concept or skill has been mastered, through exploration, clarification, practice and application over time, a person can represent it in multiple ways, has the mathematical language to be able to communicate related ideas, and can think mathematically about the concept so that they can independently apply it to a totally new problem in an unfamiliar situation.

The Concrete-Pictorial-Abstract (C-P-A) approach:

Mathematical understanding is developed through the use of representations that are first of all concrete (e.g. Base 10 resources, counters), then pictorial (e.g. array, images) to then facilitate abstract working (e.g. columnar addition, long multiplication). It is important that conceptual understanding, supported by the use of representation, is secure for procedures and if at any point a pupil is struggling with a procedure, they should revert to concrete and/or pictorial resources and representations to solidify understanding. This policy aims to show examples of concrete and pictorial representations alongside abstract wherever possible.

Aims of this policy:

- To ensure consistency and progression in our approach to calculation
- To ensure that children use the C-P-A approach to develop their mathematical understanding of the four operations
- To ensure children can use these methods confidently and accurately

1. Stages in Addition:

Addition – Early Stages:

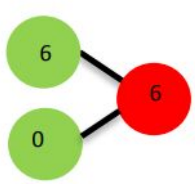
One of the Early learning Goals for Reception children is Number.

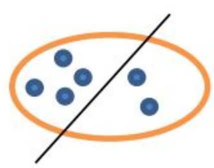
Children should be able to count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

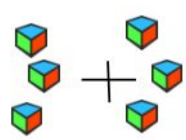
Pupils must be provided with opportunities to develop their skills so that they are able to count reliably, including one to one correspondence and count on from a given number. Pupils should be given the opportunity to count out sets of objects and then combine them to make a total:

'You have five apples and I have three apples. How many apples altogether?'

Pupils should recognise different ways of making numbers, e.g. 6 can be made as:





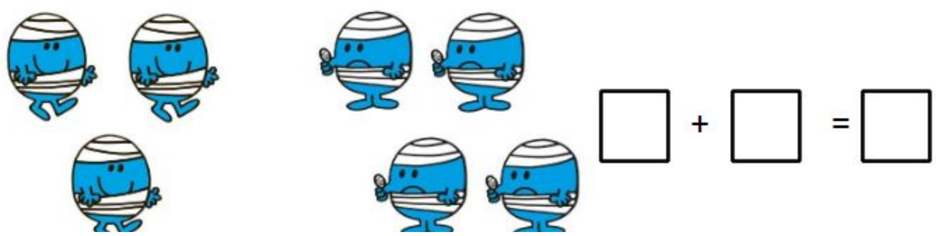


Addition – Year 1:

For the first term of Year 1, children will only work with numbers within and including 10. They will master the skills of addition involving numbers to 10 in order to make clear links when extending the numbers to 20. Throughout the year children will be introduced to numbers up to 100, this will include developing skills such as; ordering, comparing and knowledge of place value.

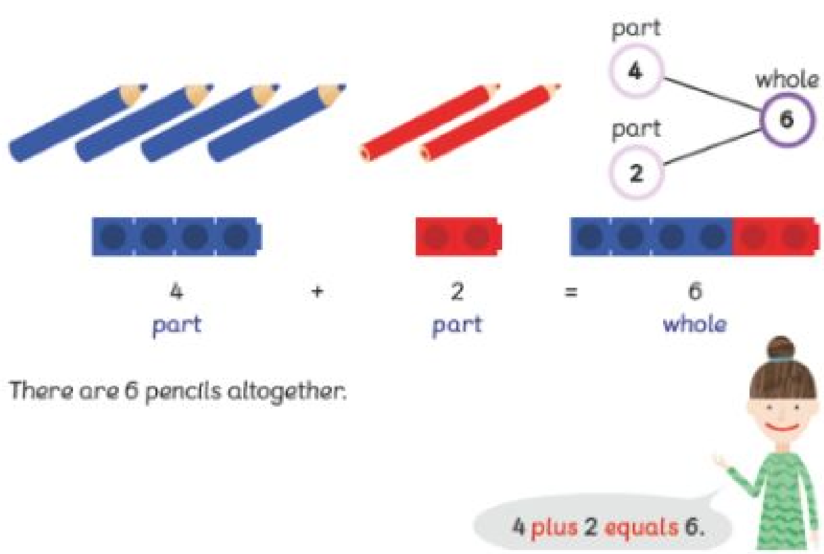
Adding using number bonds:

Children will use pictorial representations and their ability of 1:1 correspondence to decipher the calculation and find the outcome.



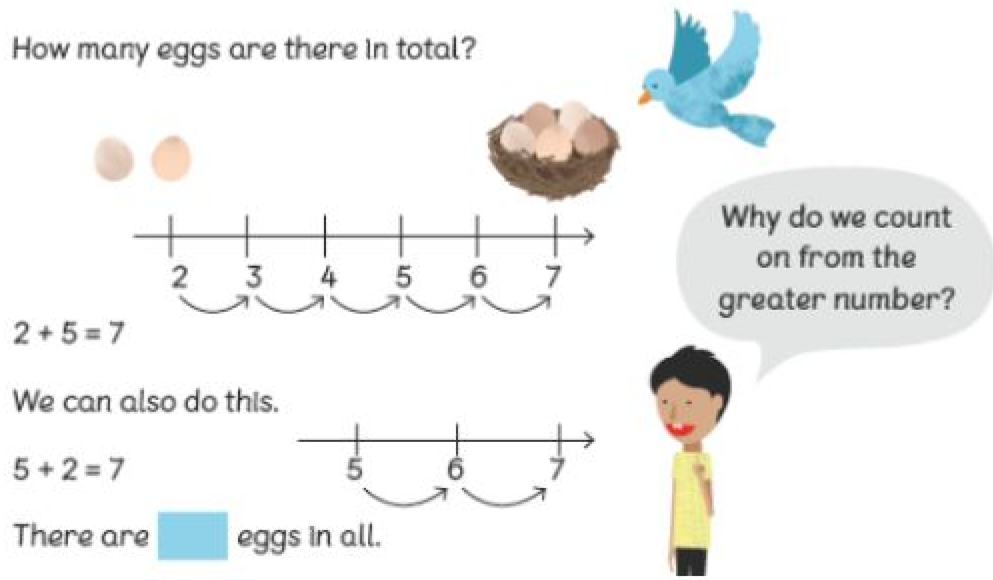
(It is important that children display the above using concrete objects in order to manipulate the two groups into one)

Children will then discover that a whole number can be made up of two parts.



Adding by counting on:

Children will now use their skill of retaining a number and counting on from any given number in order to add. The first number will be represented by its numeral therefore children should be encouraged to count on from that number.



Children will then be expected to complete the calculation and a written number sentence to demonstrate the addition. As 'adding by counting on' progresses, the scaffolding to each question is removed in order to develop fluency skills and lead on to mastery. It is important that pupils become comfortable with seeing number sentences presented in different ways, sometimes beginning with the total amount and other times with the two parts leading the way (7 = 5 + 2 and 5 + 2 = 7). Pupils should be able to explain what equals means. Their definition should include words such as 'same', 'balance' and 'equal value'. This will support the idea that there can be a plus sign on either side of the equals sign.

Making number stories:

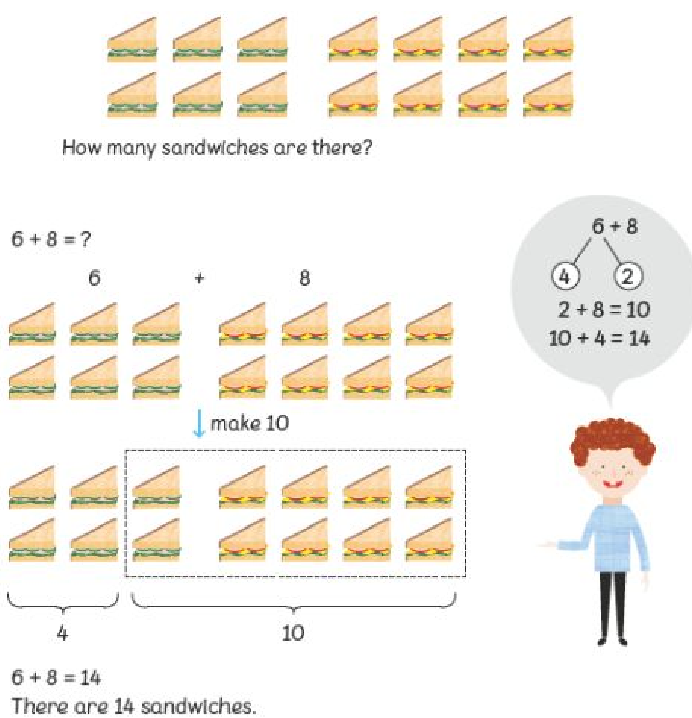
Children will now apply their knowledge of addition to number stories. These stories will enable children to put their abilities into real life contexts.

What addition stories can you make from this picture?

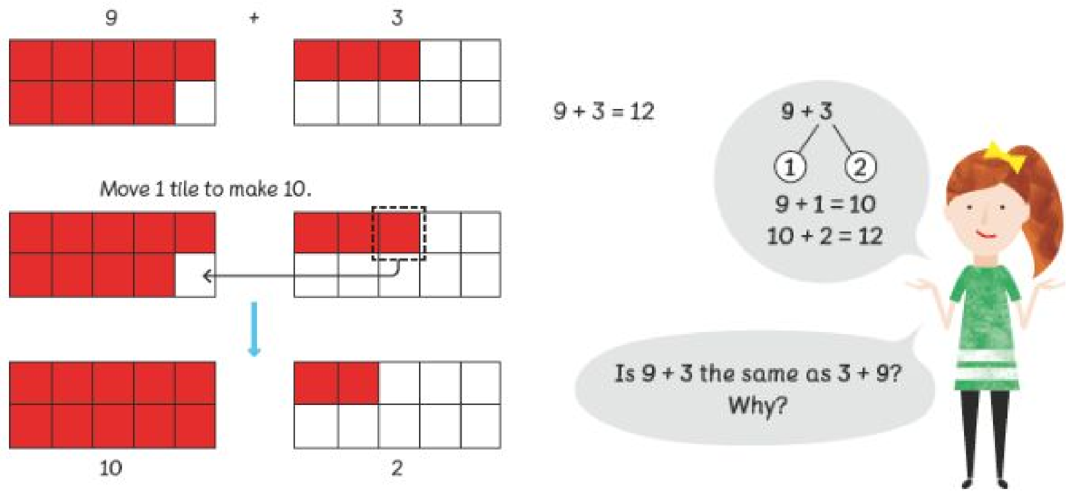
For those that need more support, they should represent the pictures with counters. They will be able to complete the boxes if read aloud to them. Children can be challenged by asking them to create their own number stories using more subtle difference, e.g. the children's: clothes, colour of clothes, hair colour etc.

Add by making ten:

When children encounter addition questions with totals beyond 10, they are taught to add two numbers by first making 10 and then adding on the remainder:



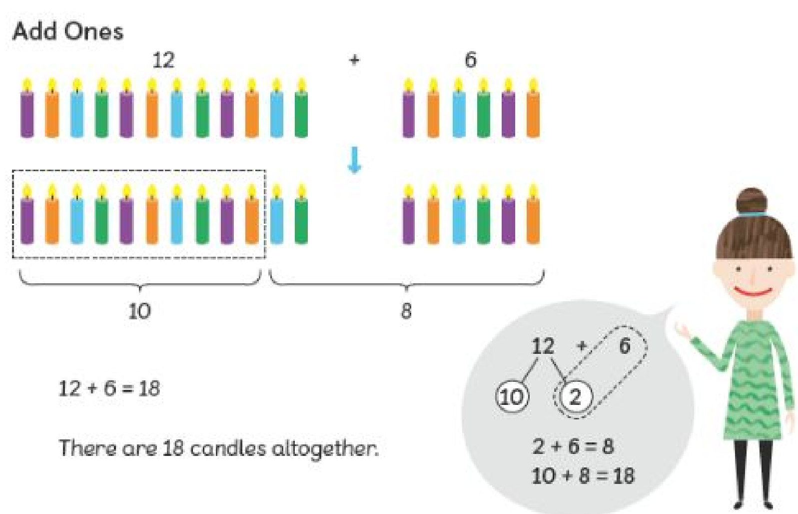
Knowledge of number bonds to 10 and the use of ten frames are very important as they allow pupils to calculate and see clearly what happens in the process.



Add by adding ones:

When adding numbers with a value greater than ten, learners are introduced to the strategy where pupils regroup numbers into tens and ones. Using ten frames, pupils are shown how 12 + 6 is just adding the ones as one of the ten frames is completely full, before developing mental calculation strategies.

How many candles are there altogether?

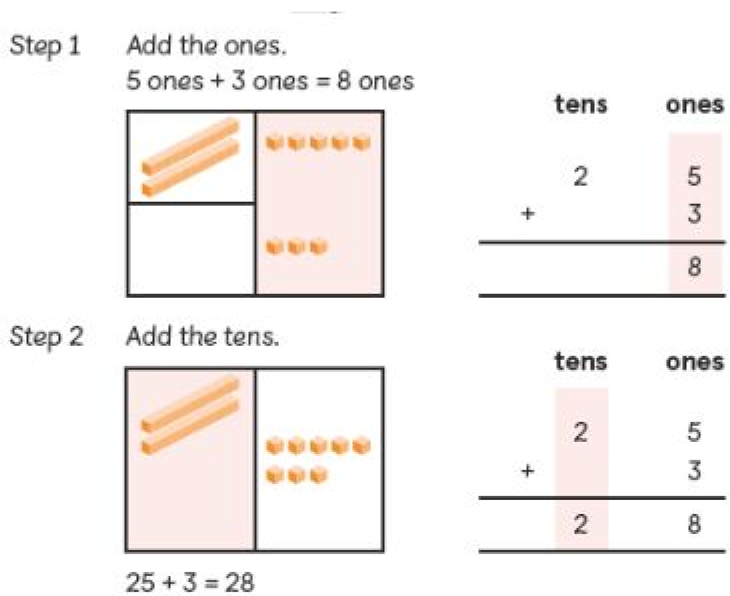


Addition – Year 2:

Having begun Year 2 learning place value of numbers to 100 in great depth, children are now prepared to build on the calculation skills learnt in Year 1. They will begin simple adding by revisiting counting on from any given number and counting on in tens from any given number.

Partitioning leading to column representation:

Pupils are now expected to add firstly, a 2-digit and one digit number, and then two 2-digit numbers, using their knowledge of place value to partition a number into its tens and ones in order to add them. They are introduced to column method of calculation; the use of place value charts, counters and base 10 resources supports their understanding of this method, eg: 25 + 3 = ?

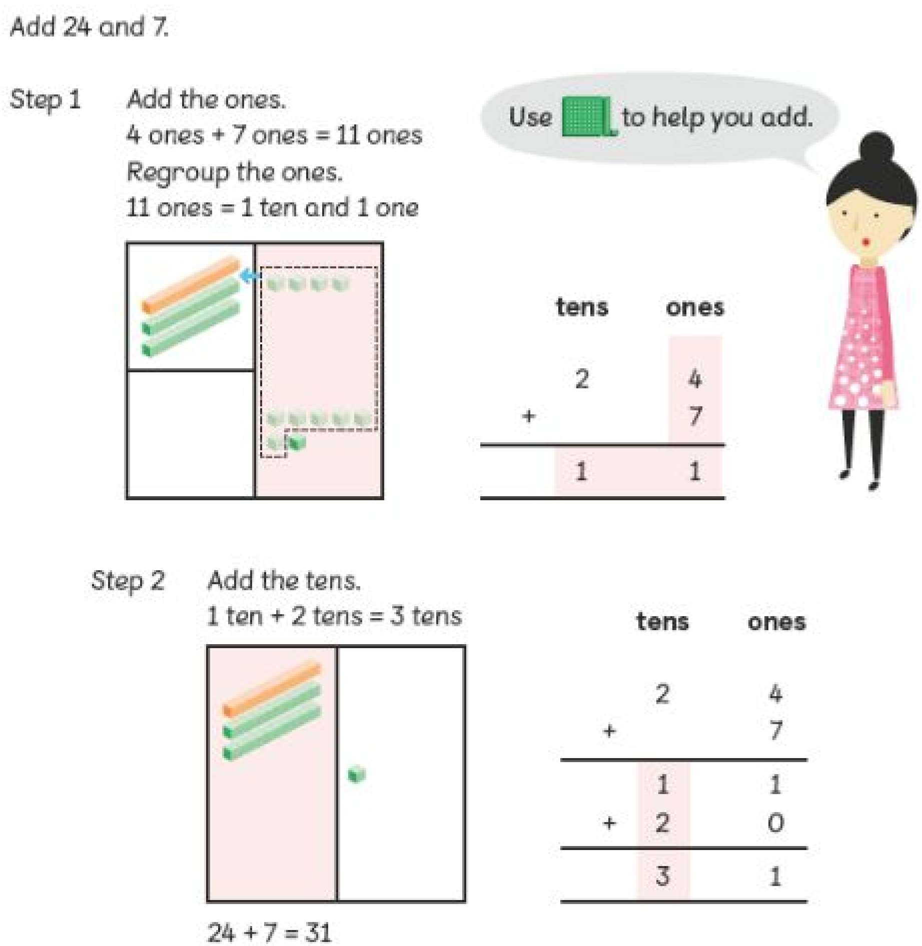


Pupils should become confident in recognising the relationship between adding ones and adding tens. Eg: 3 + 2 = 5   3 tens + 2 tens = 5 tens   30 + 20 = 50.

Once pupils are confident partitioning numbers into tens and ones, and adding them using the column method, they are ready to add numbers where the ones will total ten or more.

Adding with renaming:

Pupils are faced with the challenge of what to do when there are too many ones. When they have 10 or more ones, pupils must rename ten ones as a ten, and move it into the tens column. This is taught using Base 10 materials, then the written method where pupils are shown how to add the ones and then add the tens and then add the two sums together.



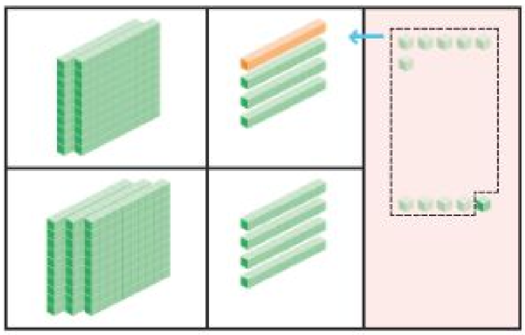
Addition – Year 3:

In year 3, pupils knowledge of place value will be extended to numbers to 1,000. As a result, pupils should be able to use mental addition strategies for adding ones, tens or hundreds to a three digit number, where there is no renaming. Building on learning from Year 2, formal written methods of calculation should be used for adding numbers with up to three digits,with renaming. Base 10 materials should be used to support pupil's conceptual understanding.

3-digit columnar addition with renaming:

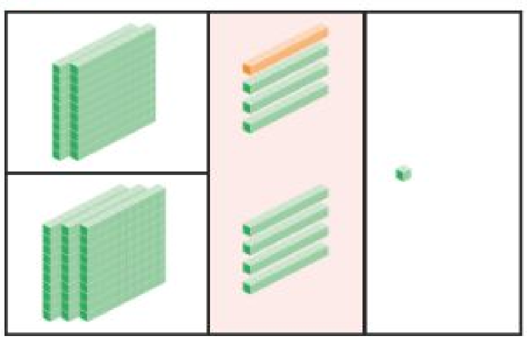
Add 236 and 345.

Step 1 Add the ones.
6 ones + 5 ones = 11 ones
Regroup the ones.
11 ones = 1 ten + 1 one



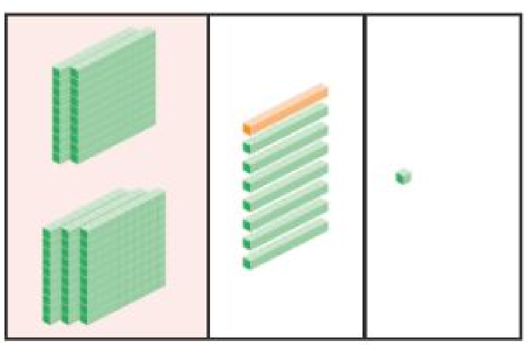
	h	t	o
		1	
	2	3	6
+	3	4	5
			1

Step 2 Add the tens.
1 ten + 3 tens + 4 tens = 8 tens



	h	t	o
		1	
	2	3	6
+	3	4	5
		8	1

Step 3 Add the hundreds.
2 hundreds + 3 hundreds = 5 hundreds



	h	t	o
		1	
	2	3	6
+	3	4	5
	5	8	1

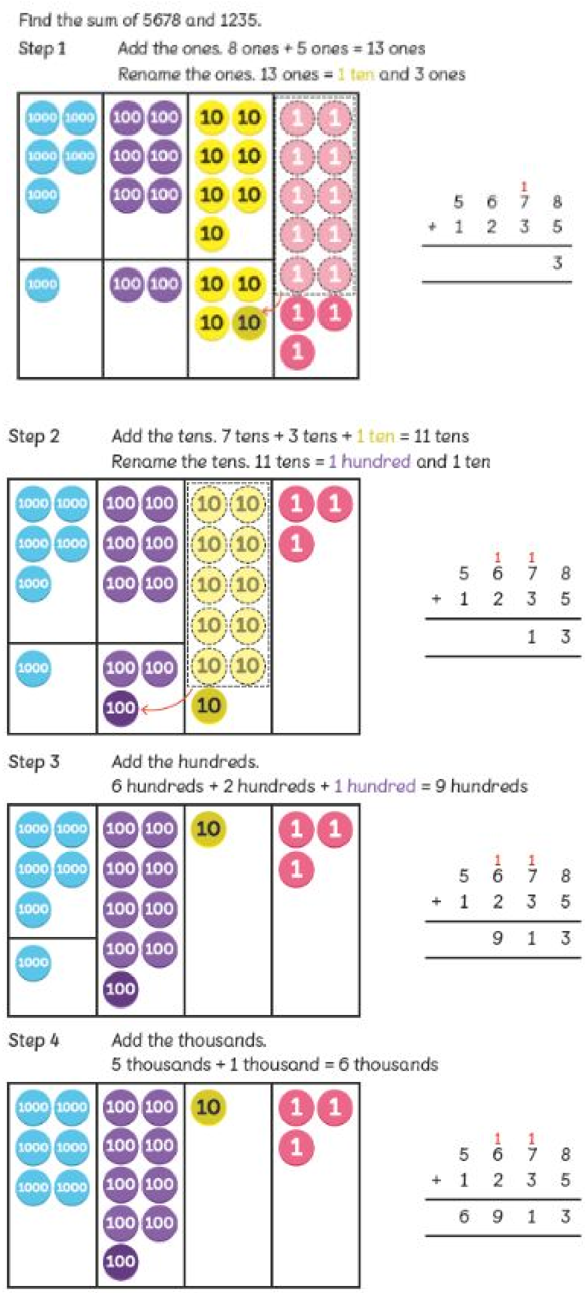
236 + 345 = 581

Pupils are taught to be able to rename and add in the ones, tens, hundreds, or all three columns, as is necessary.

Addition – Year 4:

In Year 4, pupils will be taught to calculate with numbers up to 10,000. They will continue to build upon their understanding of the column method for addition, as well as mental calculation methods. Base 10 materials, or place value counters should be used to support learning, where necessary.

4-digit columnar addition with renaming:

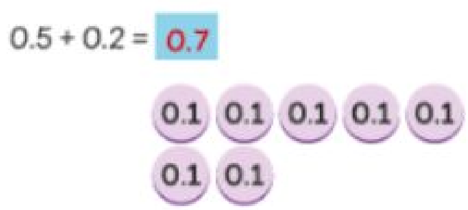


Addition – Year 5:

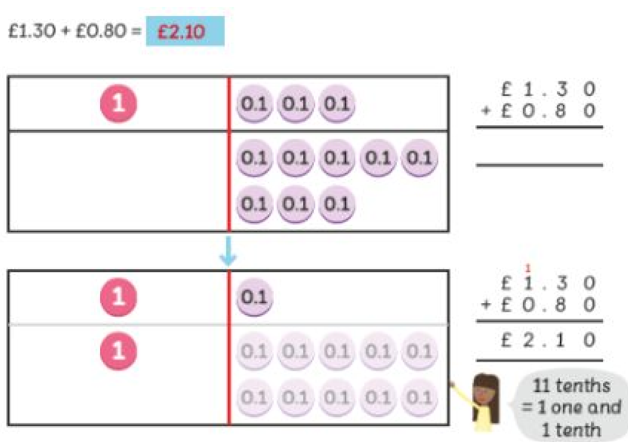
Pupils' understanding of place value is extended to numbers to 1,000,000. Formal column method for addition will be continued, as previously shown, with pupils having a secure knowledge of the place value of up to six-digit numbers.

Adding decimals:

Having revised their understanding of the value of decimals, year 5 pupils will begin calculating with them. In order to do this successfully, pupils must appreciate the similarities between calculating with decimals and calculating with whole numbers, eg: 3 + 5 = 8, 3 tenths + 5 tenths = 8 tenths, 0.3 + 0.5 = 0.8 Real life contexts should be used to support pupils' understanding of calculating with decimals, such as calculating with money or measures. Place value counters can be used to support understanding, where necessary.



Once pupils have a secure understanding of the place value of decimals, they move onto calculating decimal numbers with regrouping. Pupils are using the column method to add and subtract decimal numbers, setting out their work in the appropriate manner. They will need to understand the importance of where to place the decimal when using the column method.



Addition – Year 6:

No objectives have been included in the programmes of study explicitly related to written methods for addition in Y6. However, there is an expectation that children will continue to practise and use the formal written method for larger numbers and decimals and use these methods when solving problems, when appropriate (see previous year's guidance for methods). Our aim is that by the end of Y6, children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

2. Stages in Subtraction:

Subtraction – Early Years:

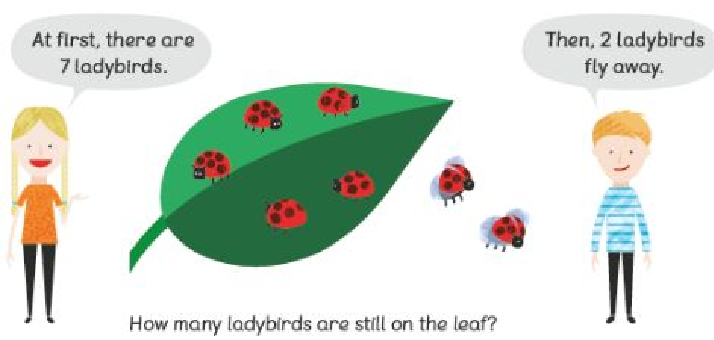
Children will engage in a variety of counting songs and rhymes and practical Activities. In practical activities and through discussion they will begin to use the vocabulary associated with subtraction. They will find one less than a given number. They will begin to relate subtraction to 'taking away' using objects to count how many are left after some have been taken away, e.g. 6 – 2 = 4

We have six apples. If we take two apples away, how many are left?

Children will also begin to count back from a given number.

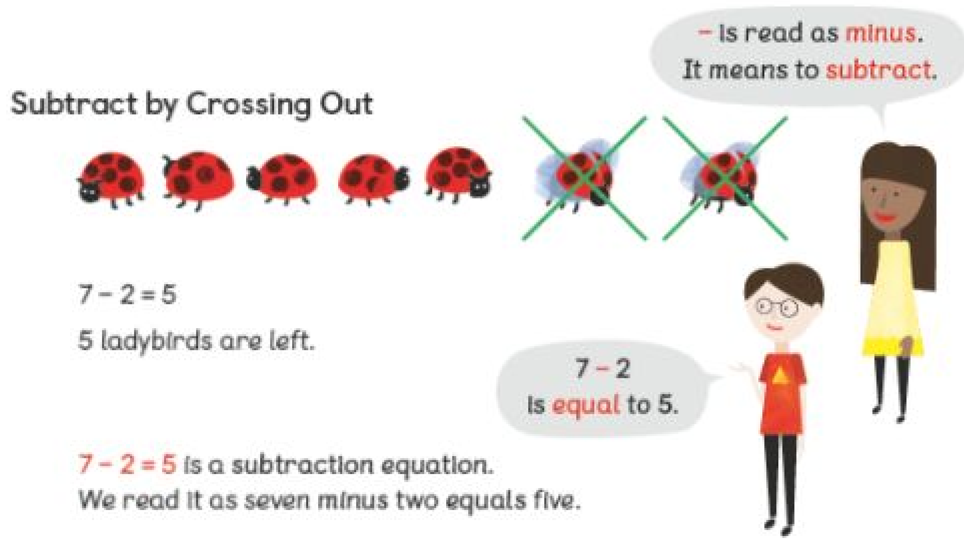
Subtraction – Year 1:

Just like addition, children will begin to subtract using numbers to 10 and move on to using larger numbers once they have mastered the skill of subtraction within 10. They will continue to use manipulatives and pictorial representations to support their understanding and use vocabulary appropriately. Children will begin subtraction by using concrete objects. They will be presented with a quantity and asked how many they would have left if they took some away. Children will physically move the number of objects away in order to see how many are left.



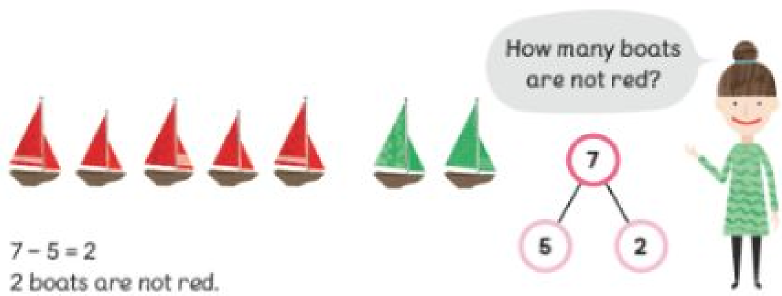
Subtraction by crossing out:

A quantity will now be represented with pictures. Children will use the skill of 'removing' a given amount of concrete objects by crossing out.



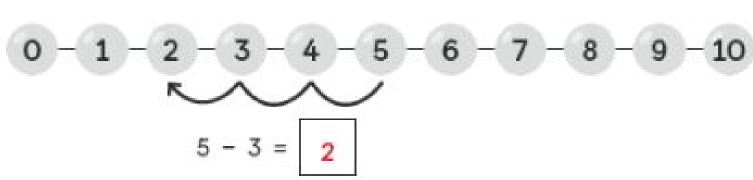
Subtraction using number bonds:

Pupils can identify the two parts that make up the whole in a number bond diagram and use it to solve a subtraction problem.



Subtraction by counting back:

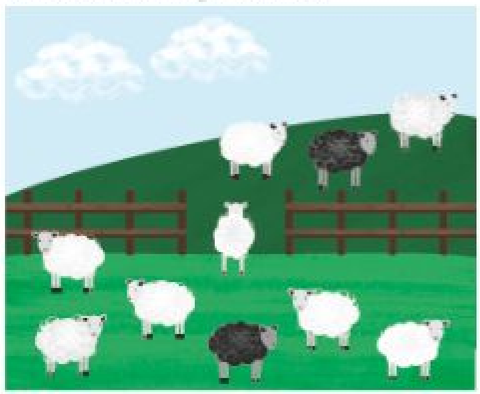
Pictorial representations will then be removed and children will learn to count backwards on a number line:



Making subtraction stories:

Pupils will now apply their knowledge of subtraction to number stories. These stories will enable children to put their ability into real life contexts.

Write the missing numbers.



(a) There are 10 sheep.
☐ sheep are black.
10 – ☐ = ☐
☐ sheep are white.

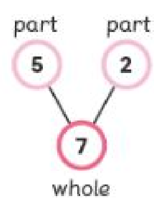
(b) There are ☐ sheep.
If ☐ sheep walk away,
☐ sheep remain.
☐ – ☐ = ☐

Making a 'family' of addition and subtraction facts:

Pupils will begin to understand that subtraction is the inverse of addition, and be able to create a 'family of facts' involving addition and subtraction, using picture and concrete materials.

How many apples are there altogether?

$\boxed{5 + 2 = 7}$ or $\boxed{2 + 5 = 7}$



How many apples are red?

$\boxed{7 - 2 = 5}$

How many apples are green?

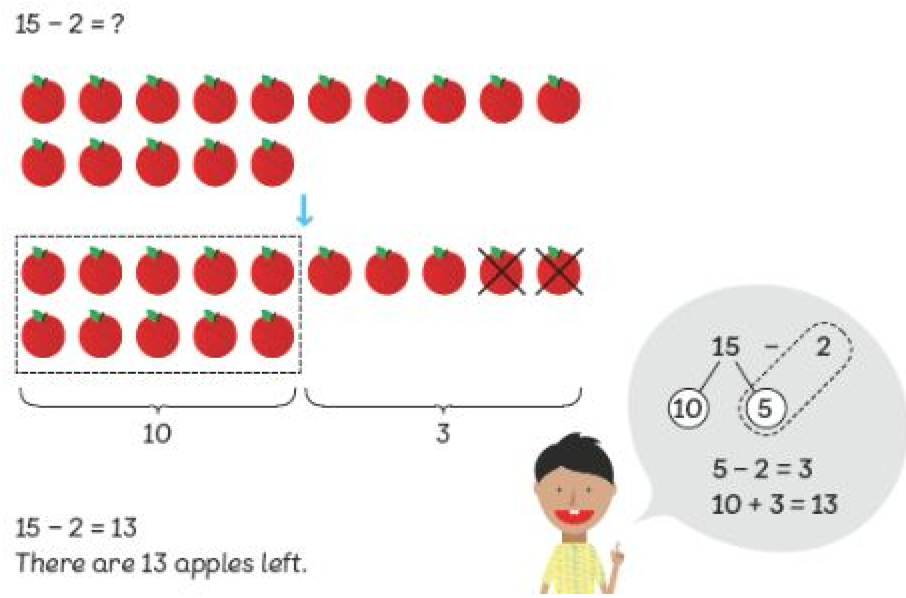
$\boxed{7 - 5 = 2}$

These are addition and subtraction equations.
They make up a family of addition and subtraction facts.

Once pupils are confident subtracting within 10, they will move onto subtraction within 20. They will continue to use previous methods, such as counting back, but also learn to use place value knowledge to support their ability to subtract.

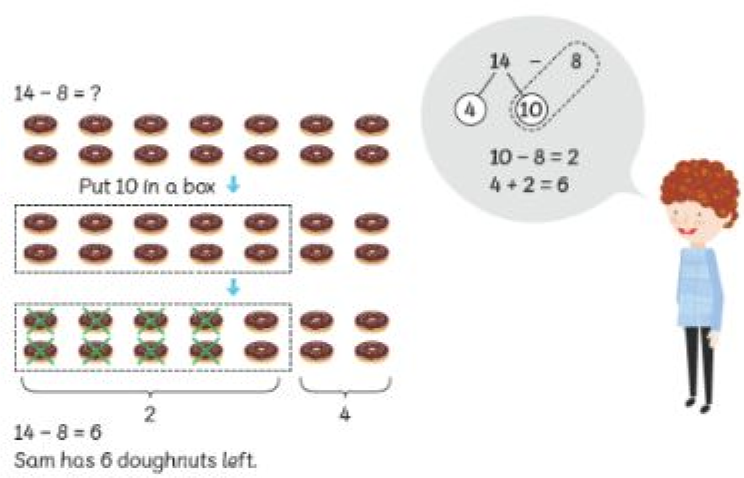
Subtract by counting ones:

Pupils will be taught how to subtract from a 2-digit number by only subtracting from the ones column; they should be able to partition a number into tens and ones, using 'part, part, whole' to use this method.



Subtract from 10:

This method of subtraction involves subtracting from the tens rather than from the ones. Using ten frames for this activity is very useful as pupils will be able to see what is happening in this concept. It is a useful method when there are not enough ones to subtract from (as shown in the method above).

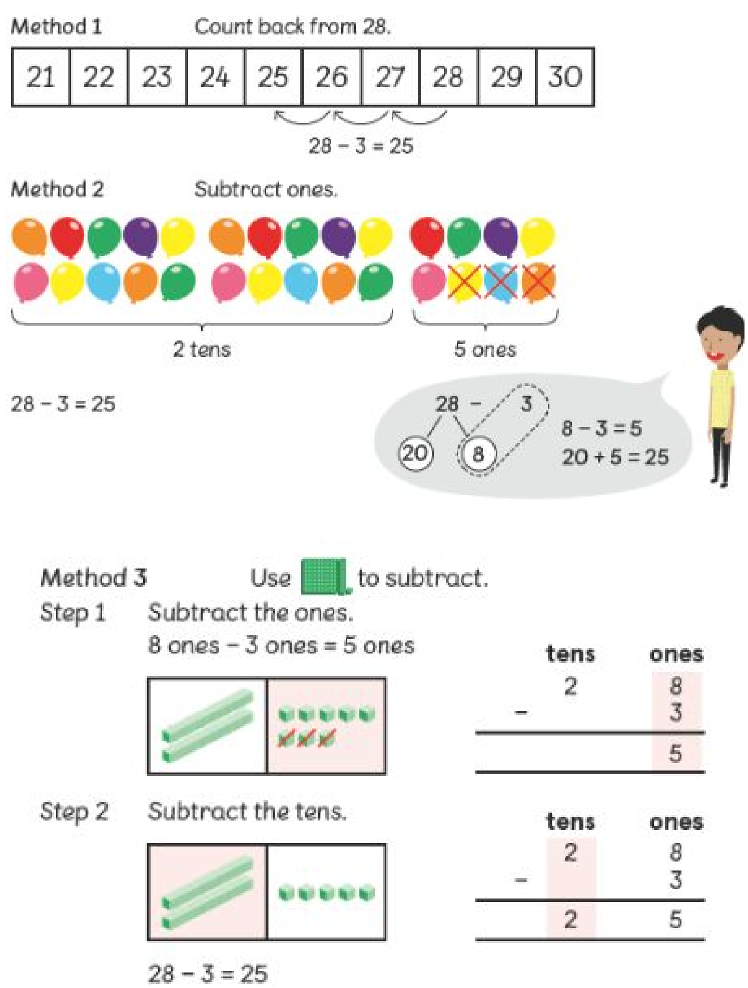


Subtraction – Year 2:

Children will begin Year 2 by recapping methods learnt in Year 1.

Partitioning leading to column representation – subtracting ones:

While pupils will be taught how to subtract using a formal written method, the column method, they will still be encouraged to use previously learned methods as is appropriate – e.g. crossing out, counting back, subtracting the ones. The move towards using the written column method is to lead pupils towards the most efficient way of solving calculations with more complex numbers, and build upon their knowledge of using the column method for addition. This method should still be supported by the use of base 10 manipulatives, e.g. 28 −3



Partitioning leading to column representation – subtracting tens:

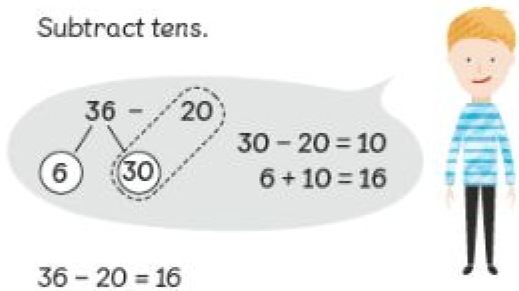
Pupils should be confident in recognising the relationship between subtracting ones and subtracting tens, e.g. 6 – 4 = 2, 6 tens – 4 tens = 2 tens, 60 – 40 =20

Subtract 20 from 36.

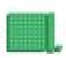
Method 1 Count back in tens from 36.

36 − 20 = 16

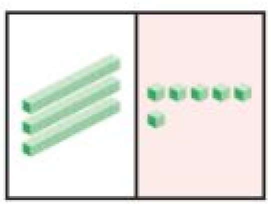
Method 2 Subtract tens.



36 − 20 = 16

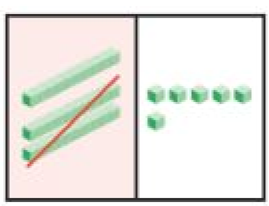
Method 3 Use  to subtract.

Step 1 Subtract the ones.



	tens	ones
	3	6
−	2	0
		6

Step 2 Subtract the tens.
3 tens − 2 tens = 1 ten

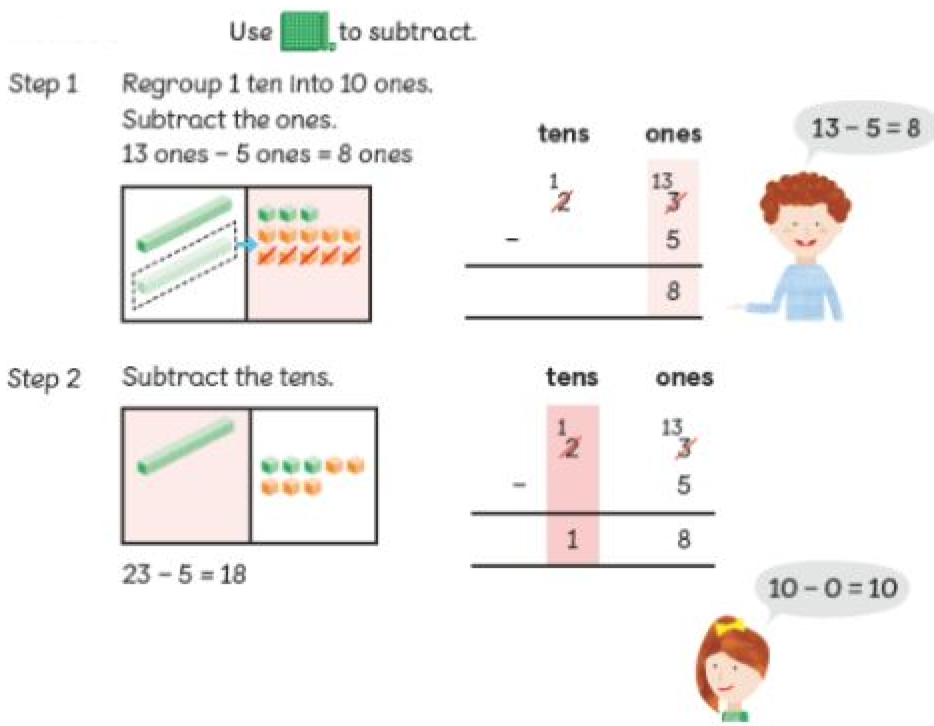


	tens	ones
	3	6
−	2	0
	1	6

36 − 20 = 16

Partitioning leading to column representation – subtracting with renaming:

Pupils are taught that when faced with having to subtract ones from a lesser amount of ones , they will need to regroup a ten to make ten ones in order to do so (place value charts and Base 10 materials are necessary to support understanding at this stage), e.g. 23 – 5



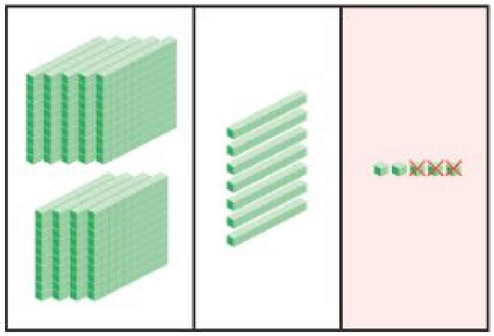
Pupils will finally learn how to subtract a 2-digit number from another 2-digit number, with renaming, using this method.

Subtraction – Year 3:

Pupils in Year 3 begin the year learning about place value of numbers to 1000; as a result of this growing understanding, they will learn how to calculate with 3-digit numbers, building upon their knowledge of columnar subtraction and mental strategies learned in Year 2. Prior to using formal written methods, pupils learn how to use mental strategies to subtract ones, tens and hundreds from 3-digit numbers, where there is no renaming required. The next stage is to be able to subtract two 3-digit numbers without renaming, e.g. 975 – 723:

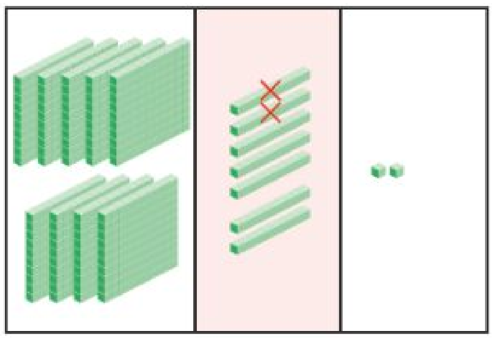
Subtract 723 from 975.

Step 1 Subtract the ones.
5 ones − 3 ones = 2 ones



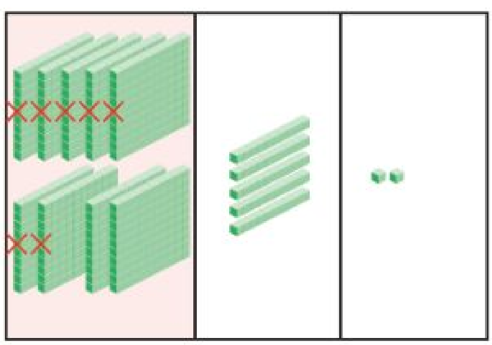
	h	t	o
	9	7	5
−	7	2	3
			2

Step 2 Subtract the tens.
7 tens − 2 tens = 5 tens



	h	t	o
	9	7	5
−	7	2	3
		5	2

Step 3 Subtract the hundreds.
9 hundreds − 7 hundreds = 2 hundreds



	h	t	o
	9	7	5
−	7	2	3
	2	5	2

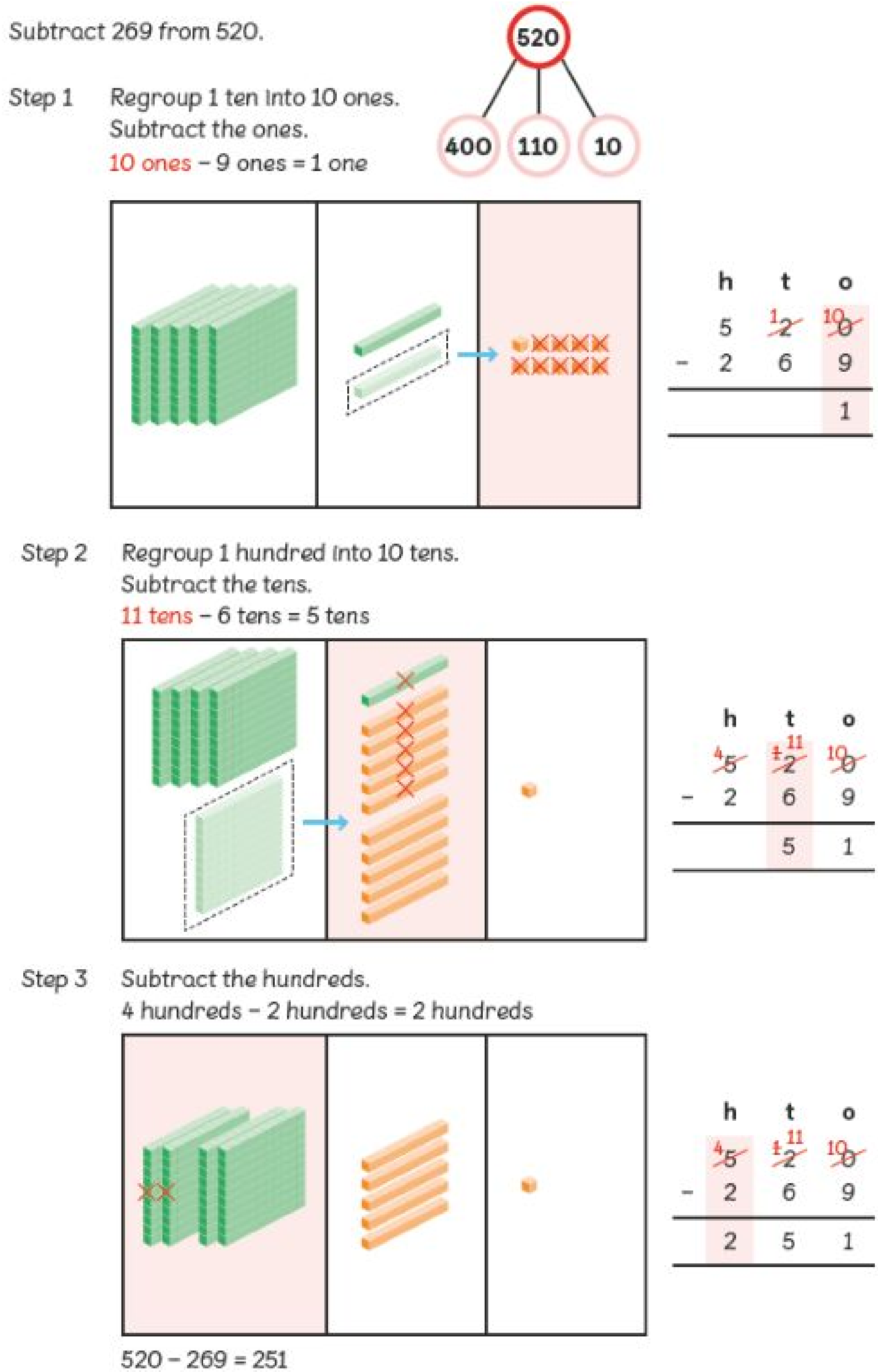
975 − 723 = 252

There were 252 beads left in the jar.

As always, the use of Base 10 materials is necessary to support understanding here, and as numbers become increasingly complex.

Subtracting with renaming:

The next stage for pupils is subtraction with renaming, where they will have to regroup 1 ten into 10 ones, or 1 hundred into 10 tens, or both.

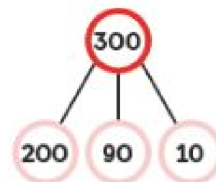


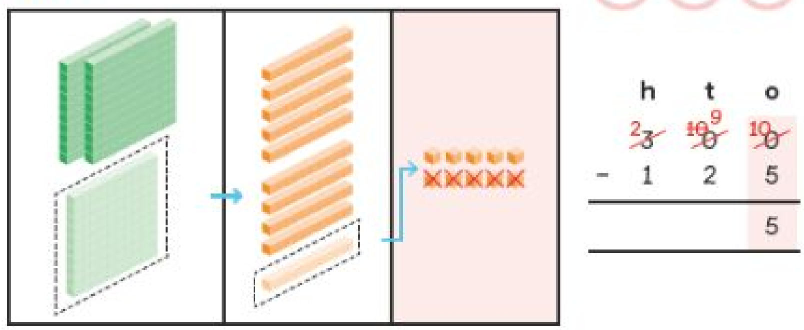
Subtracting from multiples of 100:

Pupils can easily become confused when they have to regroup twice before being able to begin a subtraction calculation. By using Base 10 materials before attempting the written calculation, it allows pupils to see the steps involved, where 1 hundred needs to be regrouped into 10 tens, before 1 ten can be regrouped into 10 ones. Once they have seen this done with the concrete resources, they should be more confident attempting the written calculation.

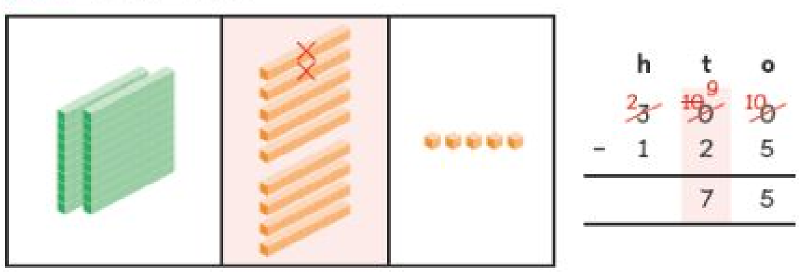
Subtract 125 from 300.

Step 1 Regroup 1 hundred into 10 tens.
Regroup 1 ten into 10 ones.
Subtract the ones.
10 ones − 5 ones = 5 ones

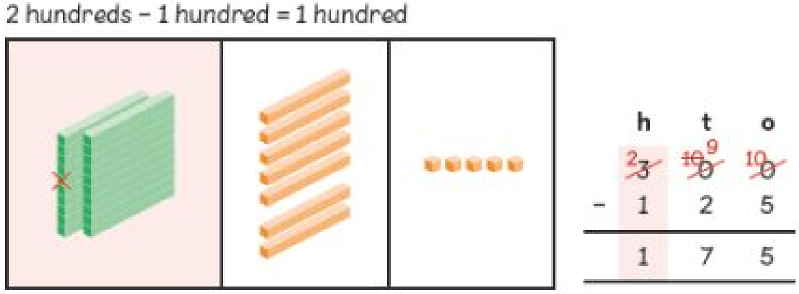




Step 2 Subtract the tens.
9 tens − 2 tens = 7 tens



Step 3 Subtract the hundreds.
2 hundreds − 1 hundred = 1 hundred



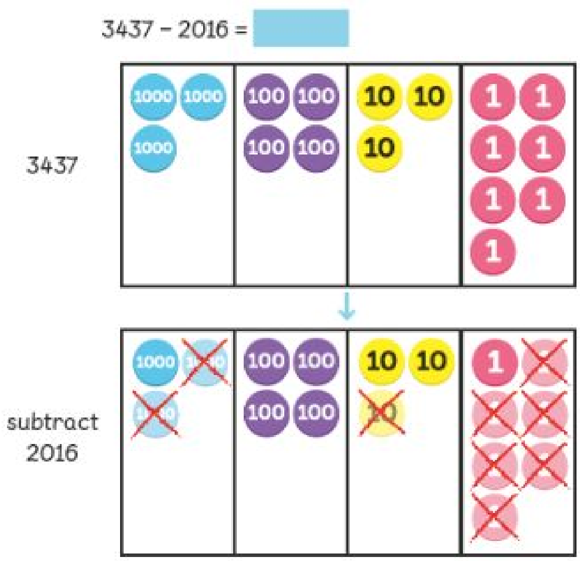
300 − 125 = 175

Subtraction – Year 4:

Pupils will have now been taught about the value of numbers up to 10,000. As a result, they will build upon their previous knowledge of mental and written strategies for subtraction, but now working up to 4-digit numbers.

Subtracting without renaming:

Where necessary, pupils should continue to use concrete resources to support their understanding of calculation, as well as building upon formal written methods, as previously learned.



$$\begin{array}{r} 3\ 4\ 3\ 7 \\ -\ 2\ 0\ 1\ 6 \\ \hline 1\ 4\ 2\ 1 \end{array}$$

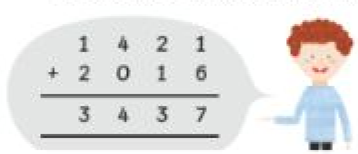
Step 1 Subtract the ones.
7 ones − 6 ones = 1 one

Step 2 Subtract the tens.
3 tens − 1 ten = 2 tens

Step 3 Subtract the hundreds.
4 hundreds − 0 hundreds = 4 hundreds

Step 4 Subtract the thousands.
3 thousands − 2 thousands = 1 thousand

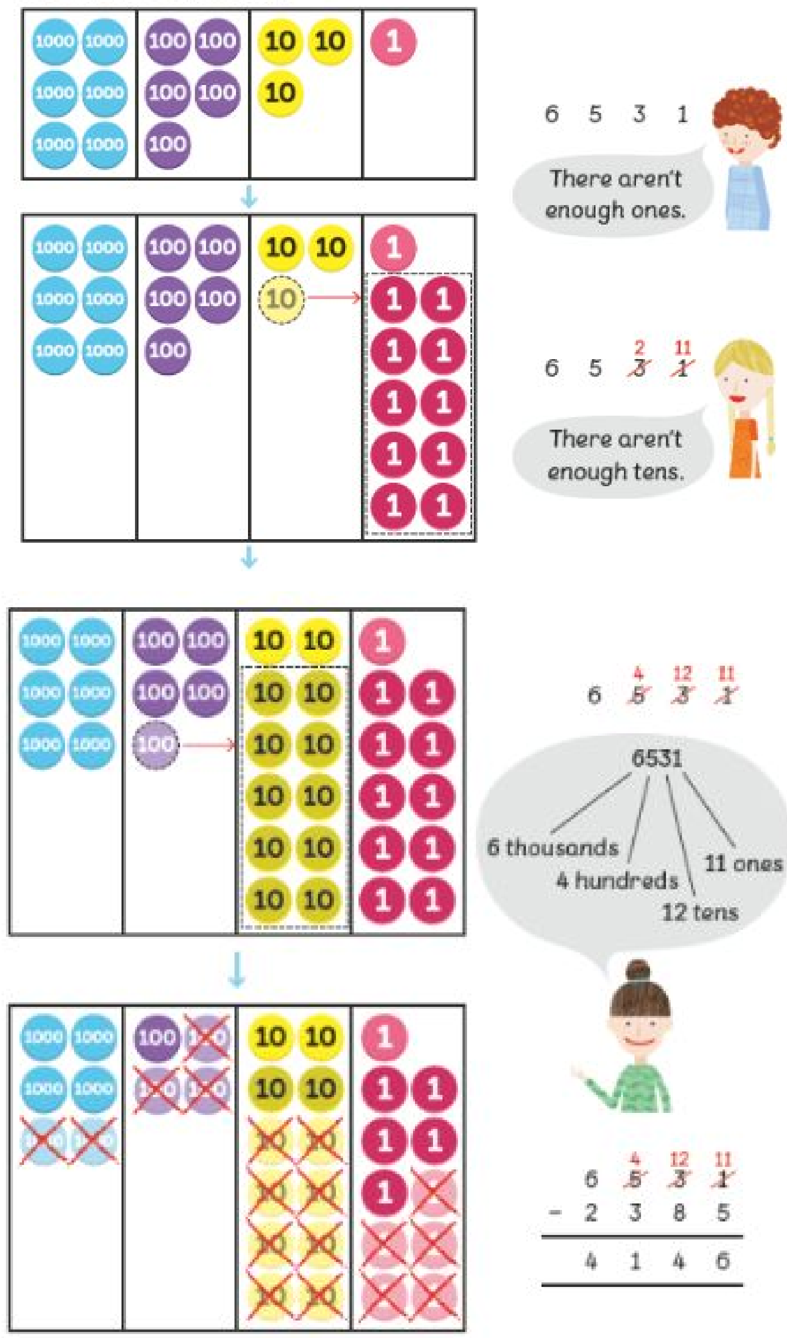
3437 − 2016 = 1421



Subtracting with renaming:

Pupils will continue to build upon their understanding of subtraction with renaming where necessary, and be taught how to subtract with renaming across the hundreds, tens and ones columns.

Subtract 2385 from 6531.



$$
\begin{array}{rcccc}
 & 6 & \cancel{5}^{4} & \cancel{3}^{12} & \cancel{1}^{11} \\
- & 2 & 3 & 8 & 5 \\
\hline
 & 4 & 1 & 4 & 6
\end{array}
$$

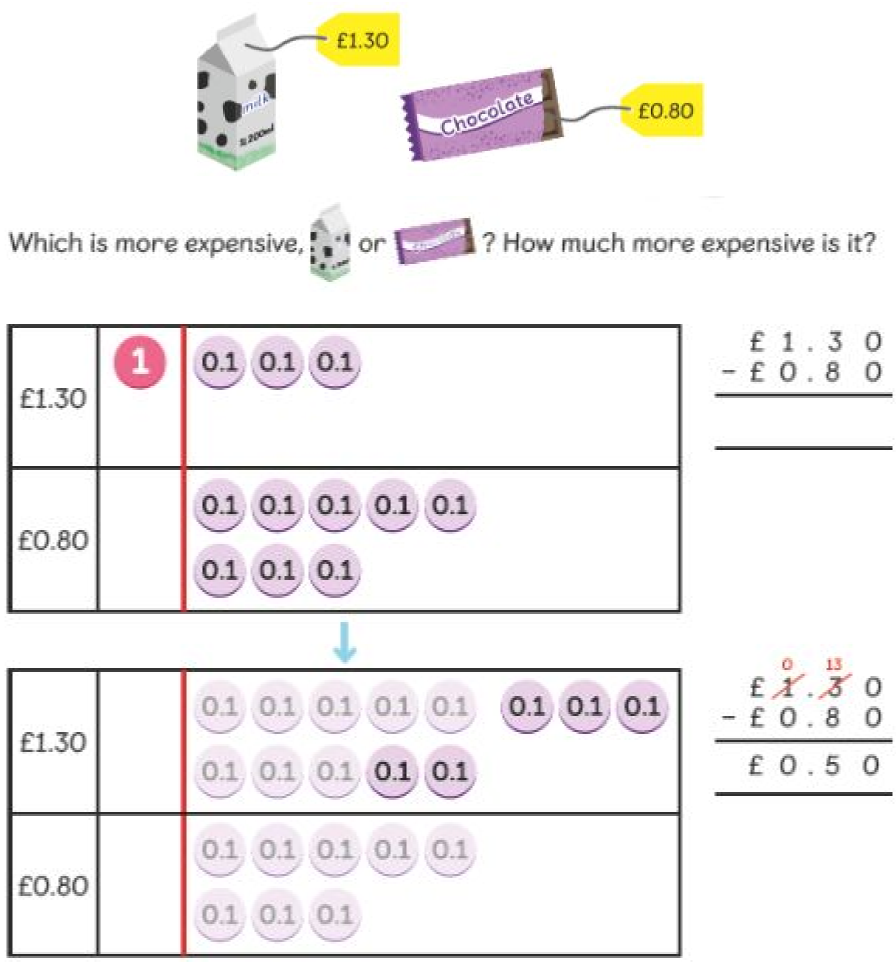
6531 − 2385 = 4146

Subtraction – Year 5:

In Year 5, pupils will be exploring subtraction of numbers to 1,000,000. They will begin by using simple strategies to subtract, such as counting back. They will then focus on subtracting within 1,000,000 using multiple key methods, such as the column method and number bonds to subtract numbers, as per guidance in previous years. Pupils will have access to concrete materials where necessary, improving their visualisation and mental skills.

Subtraction with decimals:

Pupils will continue to build upon previous learning of decimals place value, and the use of a written columnar method where appropriate in order to subtract decimals, using real life contexts.



Subtraction – Year 6:

No objectives have been included in the programmes of study explicitly related to

written methods for subtraction in Y6. However, there is an expectation that children will continue to practice and use the formal written method for larger numbers and decimals and use these methods when solving problems, when appropriate (see previous years' guidance for methods). Our aim is that by the end of Y6 children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

3. Stages in Multiplication

Multiplication – Early Years:

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving doubling.

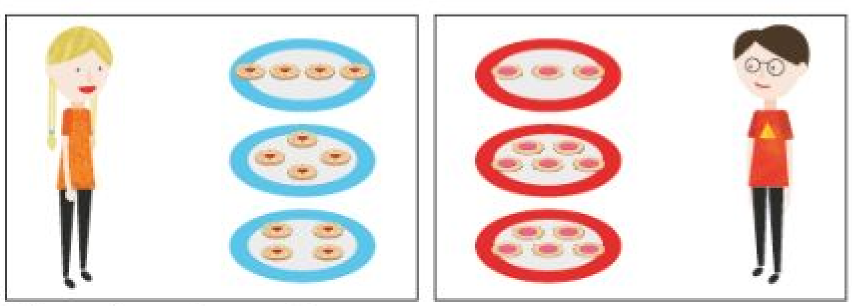
Three apples for you and three apples for me. How many apples altogether?

Multiplication – Year 1:

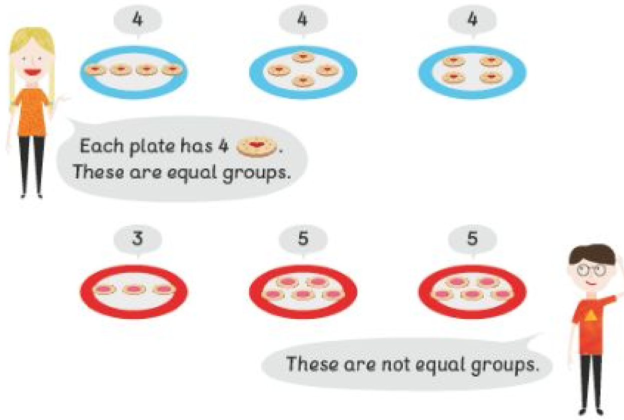
Pupils will learn the foundations of multiplication, through gaining an understanding of equal groupings, repeated addition, arrays and doubling.

Making equal groups:

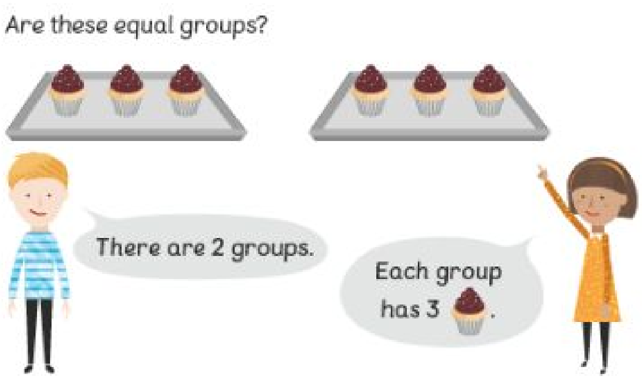
During the early stages of multiplication, children will begin to recognise and understand that multiplication is having more than one group of the same number. Therefore children will begin by deciding whether groups represented pictorially are equal groups or not. Pupils will use concrete materials to represent the problem for themselves.



Who made equal groups?

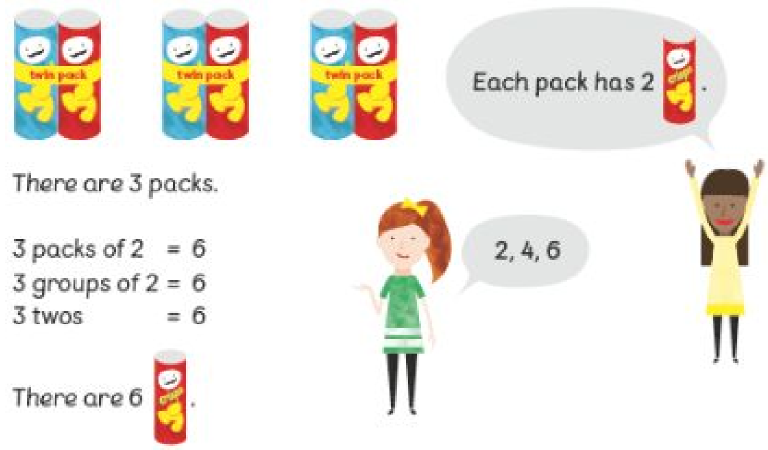


Children will then begin to put the information from the pictorial representations into sentences.



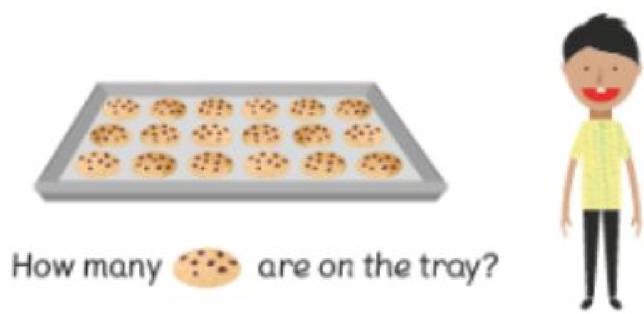
Adding equal groups:

Pupils next are taught that there are more efficient ways to count groups of the same quantity, using their previous experience of counting in twos, fives and tens.



Making equal rows:

Once pupils are confident identifying and counting equal groups, they are challenged to create equal groups themselves.



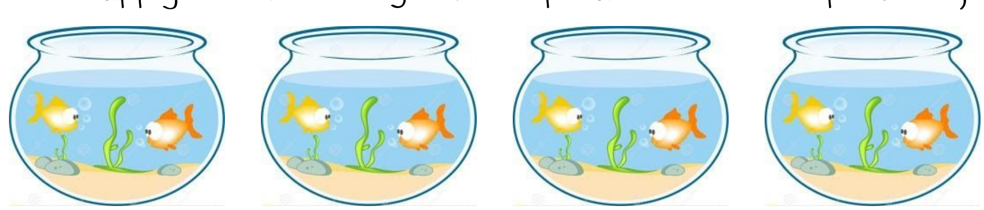
Children could create 3 rows of six, or 6 rows of three. Additional challenge can be to see how many different ways equal groups could be found.

Making doubles:

Building upon learning in the Early Years, pupils will be taught that doubling is the same as saying two groups of the same amount.

Multiplication word problems:

Pupils will next apply their knowledge of multiplication into word problems,



There are 4 fish tanks. Each tank has 2 fish. How many fish are there in all?

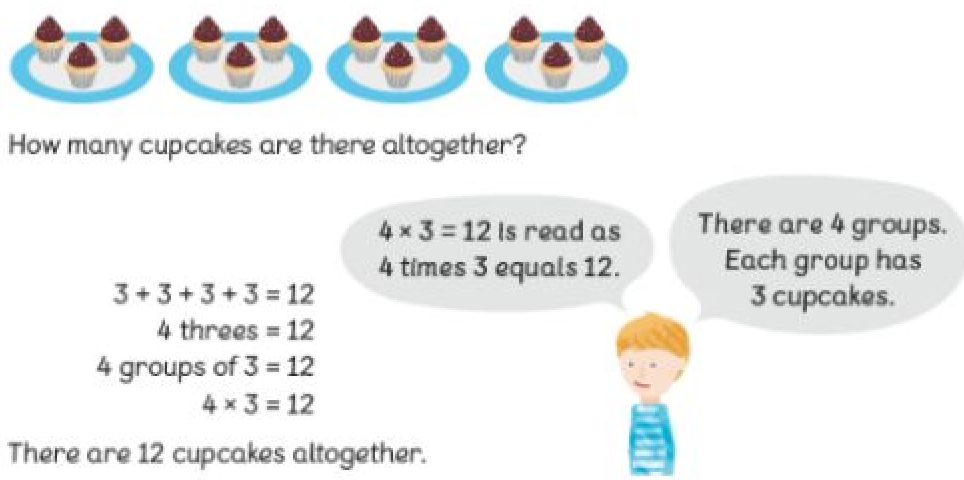
Note: Children are not expected to know the answers to multiplication questions by recall. For the above, children would not need to know that 4 x 2 = 8. Instead, they would count the fish in twos (or ones of necessary).

Multiplication – Year 2:

Children will build on the learning from Year 1 and continue to look at multiplication as equal groups. However, children will now look as the visual representations as; repeated addition, explanation in words and the calculation.

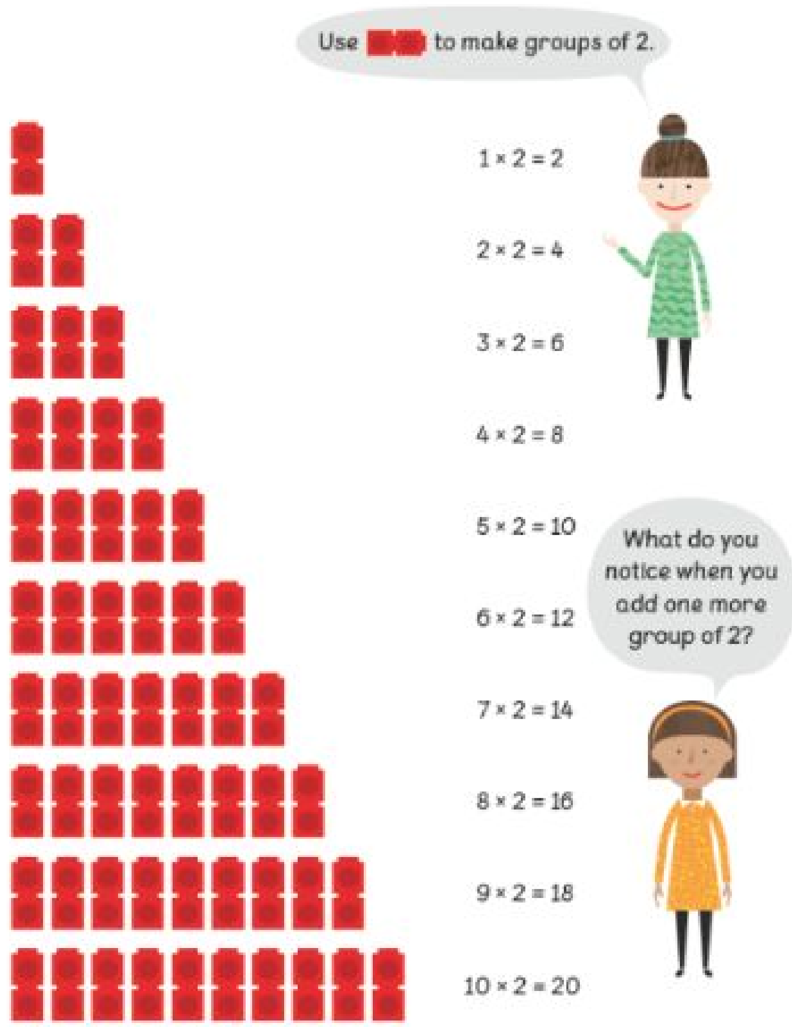
Multiplication as equal groups:

As well as consolidating that multiplication is repeated addition of equal groups and counting equal groups, pupils are introduced to the multiplication sign for the first time, as a means of writing how many equal groups there are.

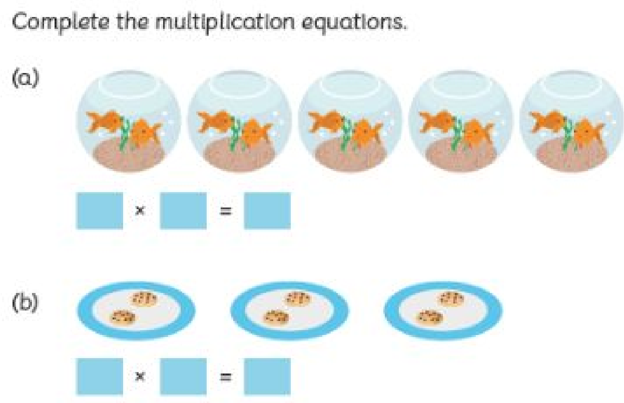


Learning of multiplication tables (2s, 5s and 10s):

Year 2 is the first year that pupils will begin to learn, recall and understand multiplication tables. They will use their knowledge of multiplication being a number of equal groups to recall times tables at speed. They will do this by seeing the patterns of each times table using concrete objects and pictorial representations. Pupils will do this by arranging concrete objects, such as cubes, into groups of 2; this way they will notice patterns and relationships between numbers each time an extra two is added.



Pupils will now revisit previous learning and apply their knowledge of the 2 times table into real-life context. Their ability to display and read the 2 times tables using concrete objects will be transferred into pictorial representations:



Finally, children will work on recalling these times tables from memory (though concrete objects can still be used to solve the equations if necessary).

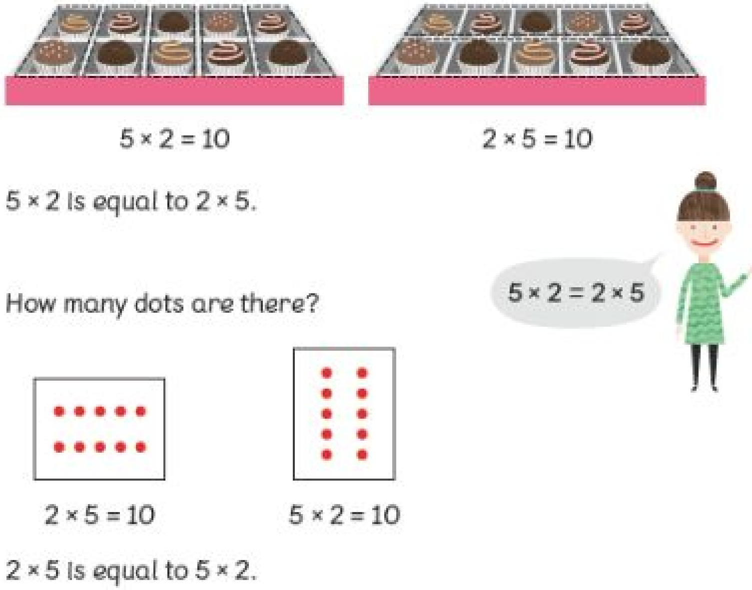
Children will learn the 2, 5 and 10 times table using the steps outlined above and are expected to know these times table by recall by the end of Year 2 (this is the order that they will be taught).

Knights of the Times Table:

From summer term in Year 2, pupils are challenged to become 'Knights of the Times Table; this is recalling each times table, out of order, in 90 seconds (this also includes the recall of the division facts).

Commutative Law:

Once pupils have been taught their times tables, they are introduced to the commutative law, whereby two numbers can be multiplied in any order; this can be shown with concrete objects, pictorial representation or the use of arrays.

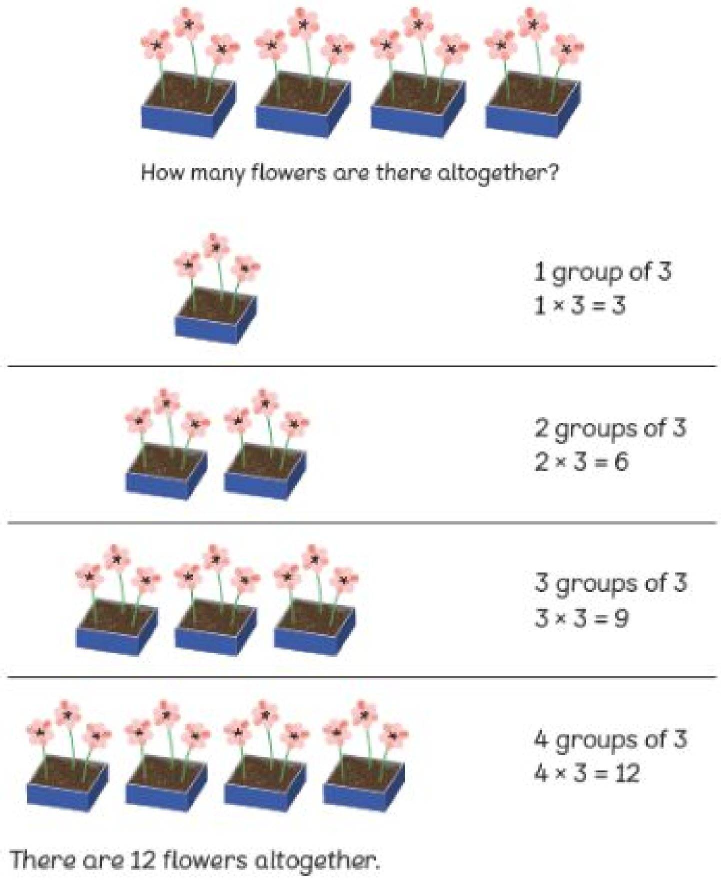


Multiplication – Year 3:

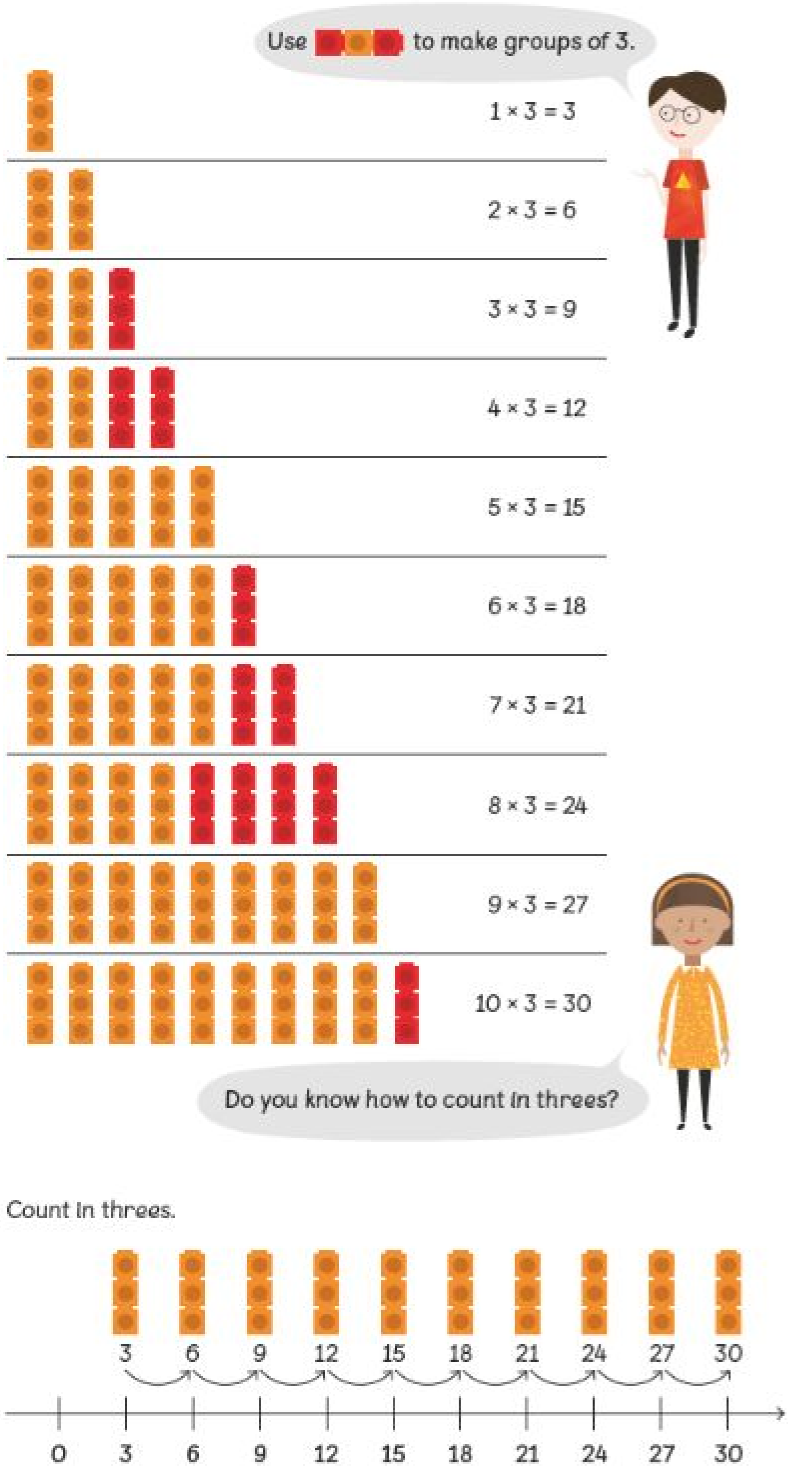
As pupils progress in Year 3, they will: recall and use multiplication facts for the 3, 4 and 8 multiplication tables; write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for 2-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

Learning of multiplication tables (3s, 4s and 8s):

Building upon their knowledge of times tables from the previous year, pupils should understand that multiplication is repeated addition of equal groups, with the most efficient method of counting these groups being to multiply them. The use of repetitive phrasing to embed the idea that we are talking about equal groups, such as 'There are 4 groups of 3. 4 equal groups of 3 make 12. 4 x 3 =12.' helps to ensure pupils' conceptual understanding of what multiplication tables mean, rather than just rote learning.

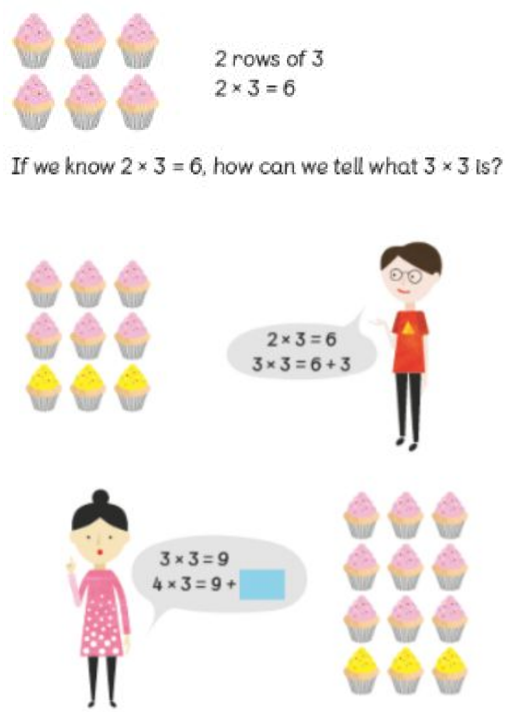


Pupils continue to use concrete resources and pictorial representations to see the patterns, as well as use their knowledge of the 2s, 5s and 10 times tables.



Using patterns to learn multiplication tables:

Pupils are encouraged to see patterns in times table to help commit them to memory easily. For example: below are 2 rows of 3 cupcakes, therefore, we write it as $2 \times 3 = 6$. Pupils are taught that they can work out 3×3 if they know $2 \times 3 = 6$ by drawing another row of 3 cupcakes beneath the first 2 rows. Now 3 rows of 3 can be written as $3 \times 3 = 9$. If pupils understand that 3×3 means 1 more group of 3, then they learn that the next number is always 3 more than the previous one. Therefore, if they know $3 \times 3 = 9$, they can find 4×3 by adding 3 to 9. Similarly, if pupils know $2 \times 3 = 6$, and they know that 4 is double 2, then they will understand that $4 \times 3 = 12$ (double 6). This strategy better enables pupils to commit multiplication tables to memory, as they will have a deeper conceptual understanding as opposed to rote learning.



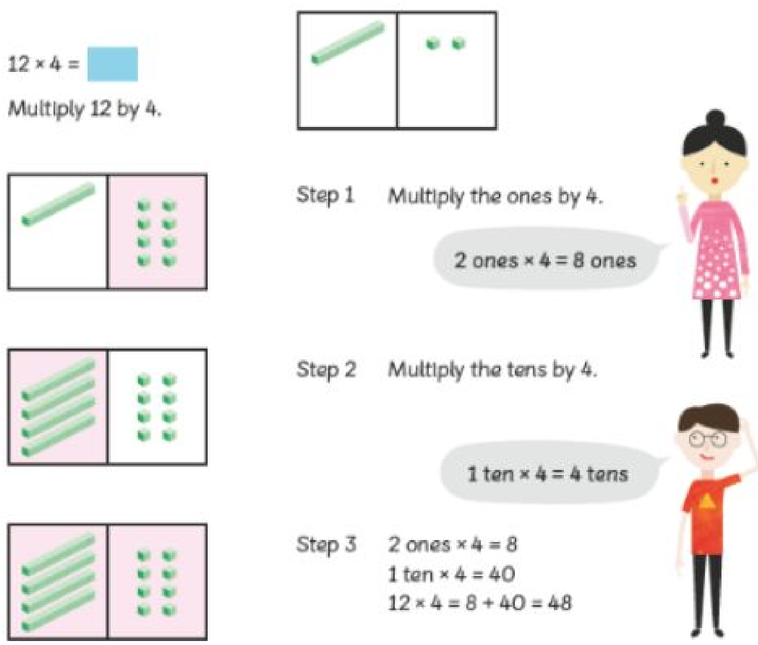
Pupils proceed to learn the 4 and 8 times tables in this manner, with emphasis been given to seeing the relationship between the 4s and 8s (pupils learn that the 8 times table is essentially doubling the 4 times table, so teaching in sequence is important) . Pupils continue to deepen their understanding of commutative law, and learn to describe multiplication as finding the product of two numbers.

Multiplying 2-digit numbers:

There are a number of steps that pupils move through in order to be able to use a formal written method for multiplying a 2-digit number; it is vital this sequence is followed in order to ensure conceptual understanding, rather than just procedural.

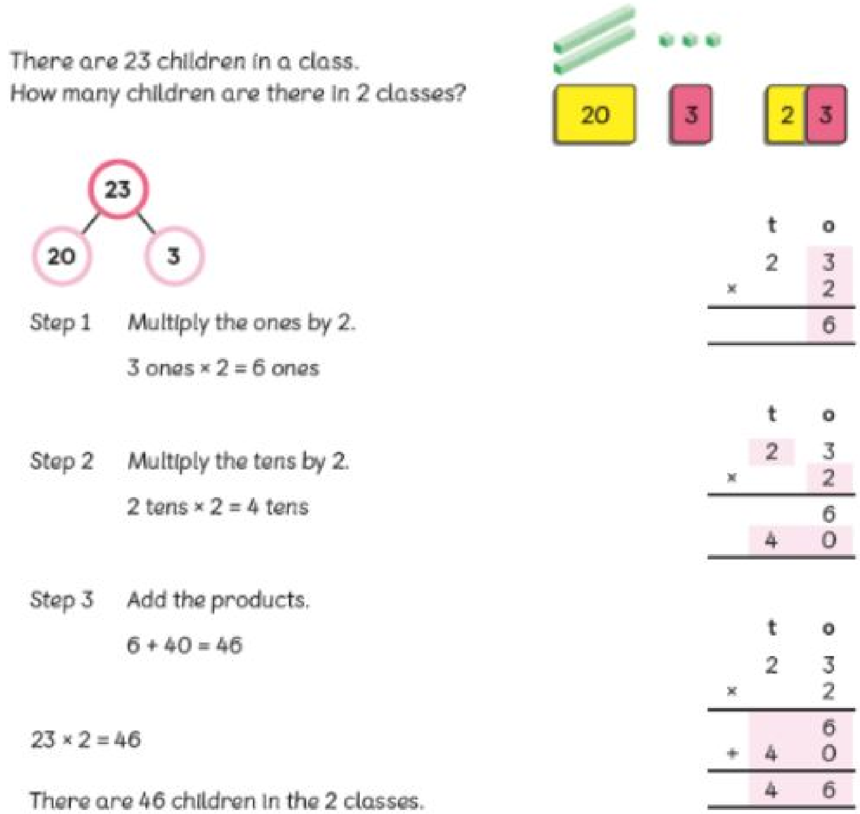
1. Pupils begin learning to multiply 2-digit numbers by seeing the relationship between multiplying ones and tens, using known facts, with Base 10 resources to support their understanding.

2. Next, pupils decompose a 2-digit number into tens and ones, and multiply each of them using known facts, then adding the answers together, as shown (the number bond method):

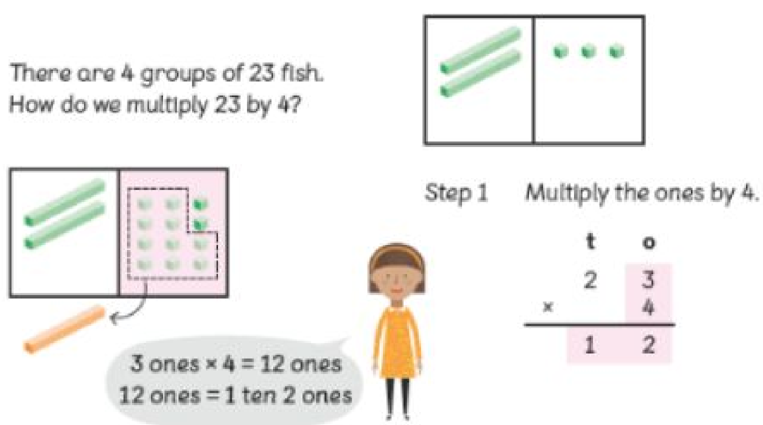


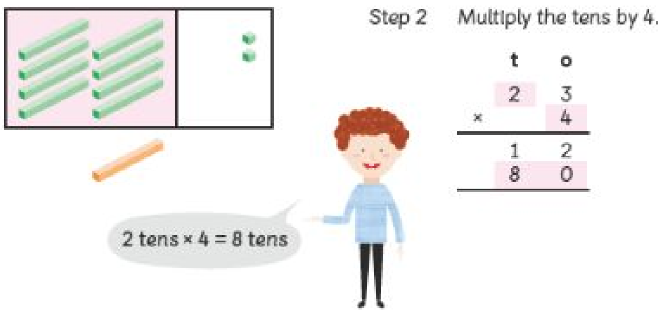
3. Pupils then move onto using an expanded version of the standard column method, ensuring that they see the connection at each step with this and the

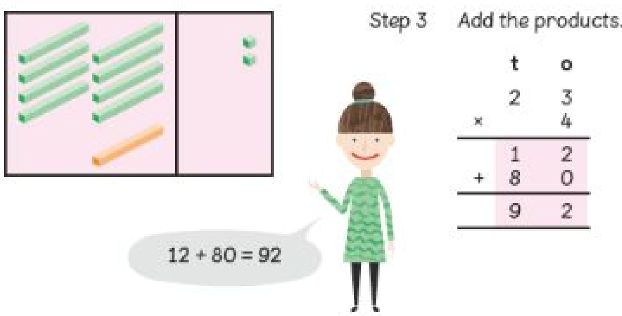
number bond method:



4. Pupils are then taught to multiply a 2-digit number using the expanded version of the column method, where regrouping is required:



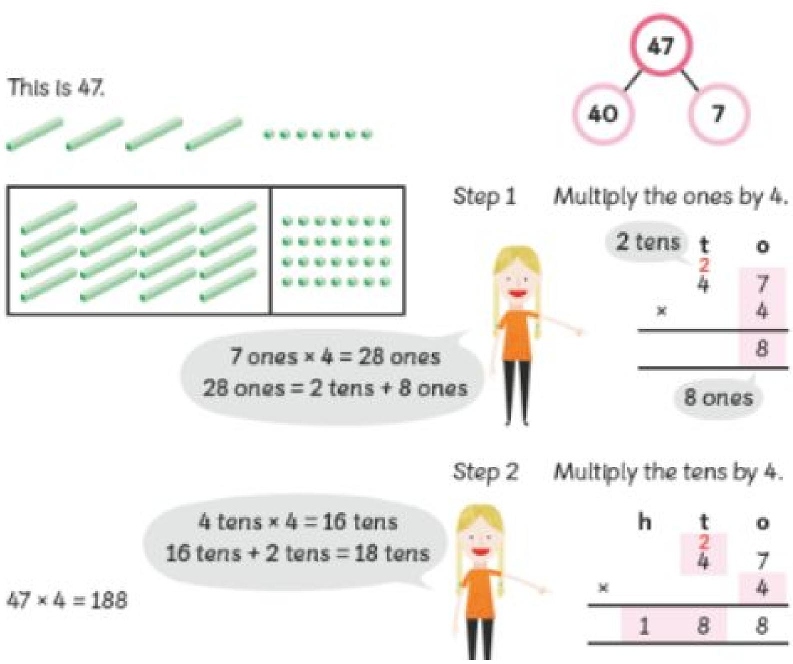




23 × 4 = 92

There are 92 fish in 4 tanks.

5. Finally, when ready, pupils are taught the standard column method, where rather than regrouping the tens in the answer box, they are added into the tens column, to save time and space (short multiplication.)



Multiplication – Year 4:

In Year 4, pupils will continue to learn their multiplication tables, learning the 6, 7, 9, 11 and 12 times tables (in that order). By the end of year 4, all pupils are expected to have a recall of multiplication and division facts for multiplication tables up to 12 x 12. Pupils will continue to learn their multiplication tables as shown in previous years. Applying conceptual understanding and procedural knowledge of multiplication will be core to pupils' ability to solve word problems.

Multiplication Tables Check (MTC):

The Multiplication Tables Check (MTC) is a Key Stage 2 assessment to be taken by pupils at the end of Year 4 (in June), from the 2019 / 2020 academic year onwards. The purpose of the MTC is to make sure times tables knowledge is at the expected level. The MTC is an online test were the pupils are asked 25 questions on times tables 2 to 12. For every question you have 6 seconds to answer and in between the questions there is a 3 second rest.

At Park, multiplication tables are practised regularly; academic clubs are offered for pupils who need additional support; weekly Knights of the Times Table challenges are completed that allow us to track progress and we use the Times Table Rock Stars website and app as a platform for children to play multiplication and division games, improve their recall and speed and play in an emulation of the MTC test. Supporting pupils' progress in knowing their times tables at home is a vital part of their learning.

Multiplying two and 3-digit numbers:

As pupils progress through Year 4, they will continue to use their tables and knowledge of place value to multiply multiples of 10, leading to the multiplication of 2-digit numbers using short multiplication (see Year 3 guidance). They will then use their knowledge of multiplying multiples of 10 when multiplying multiples of 100, leading to multiplying 3-digit numbers using short multiplication.

Multiplying multiples of 100:

Pupils begin the process of multiplying 3-digit numbers by learning how to multiply multiples of 100; they do this by using their knowledge of how to multiply multiples of 10, i.e. use a times table fact and then make it 10 times bigger.

$7 \times 300 = \square$

Method 1

$$\begin{array}{r} 300 \\ 300 \\ 300 \\ 300 \\ 300 \\ 300 \\ +\ 300 \\ \hline 2100 \end{array}$$

Method 2

$7 \times 3 = 21$

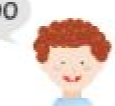
7×3 hundreds $= 21$ hundreds

$7 \times 300 = 2100$

Method 3

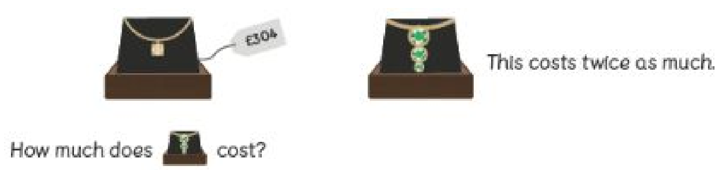
$$\begin{aligned} 7 \times 300 &= 7 \times 3 \times 100 \\ &= 7 \times 3 \times 100 \\ &= 21 \times 100 \\ &= 21 \text{ hundreds} \\ &= 2100 \end{aligned}$$

21 hundreds = 2100



Multiplying a 3-digit number without renaming:

Pupils first begin multiplication of 3-digit numbers using the number bond or expanded column method, and can use bar models (pictorial representation) or concrete manipulatives to support understanding.



$$\begin{array}{r} 300 \times 2 = 600 \\ 4 \times 2 = \quad 8 \\ \hline 304 \times 2 = 608 \end{array}$$


costs £608.

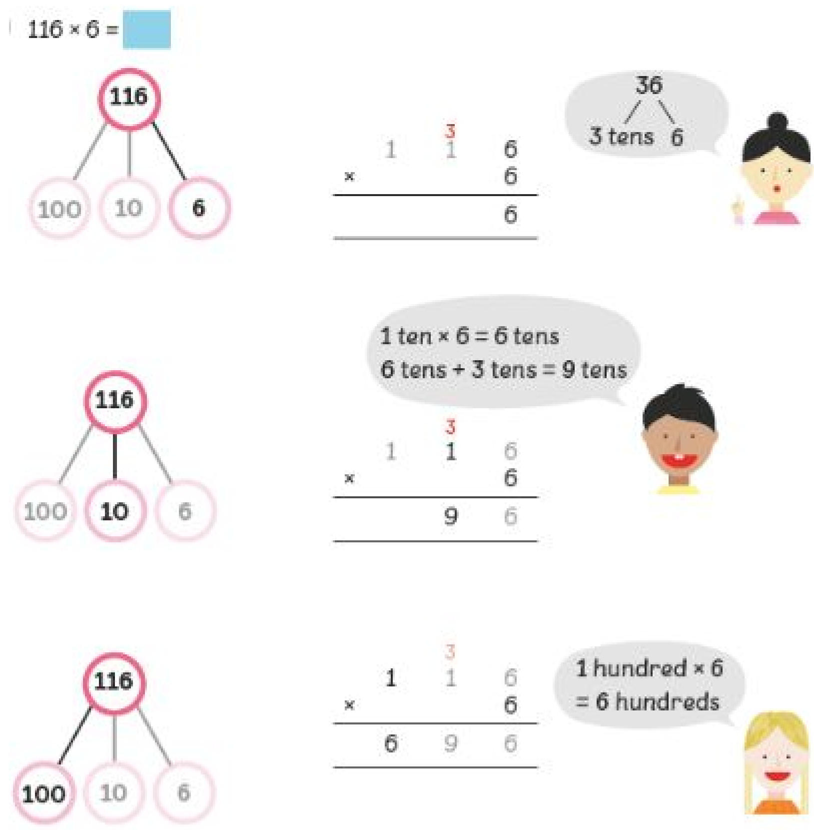
2 $304 \times 2 = \square$

$$\begin{array}{rrrr} & 3 & 0 & 4 \\ \times & & & 2 \\ \hline & & & 8 \end{array}$$

$$\begin{array}{rrrr} & 3 & 0 & 4 \\ \times & & & 2 \\ \hline & & & 8 \\ + & 6 & 0 & 0 \\ \hline & 6 & 0 & 8 \\ \hline \end{array}$$

Multiplying 3-digit numbers with renaming:

Pupils next move onto the contracted column method (short multiplication) with 3-digit numbers with renaming, shown below, as the most efficient method of completing the calculation.



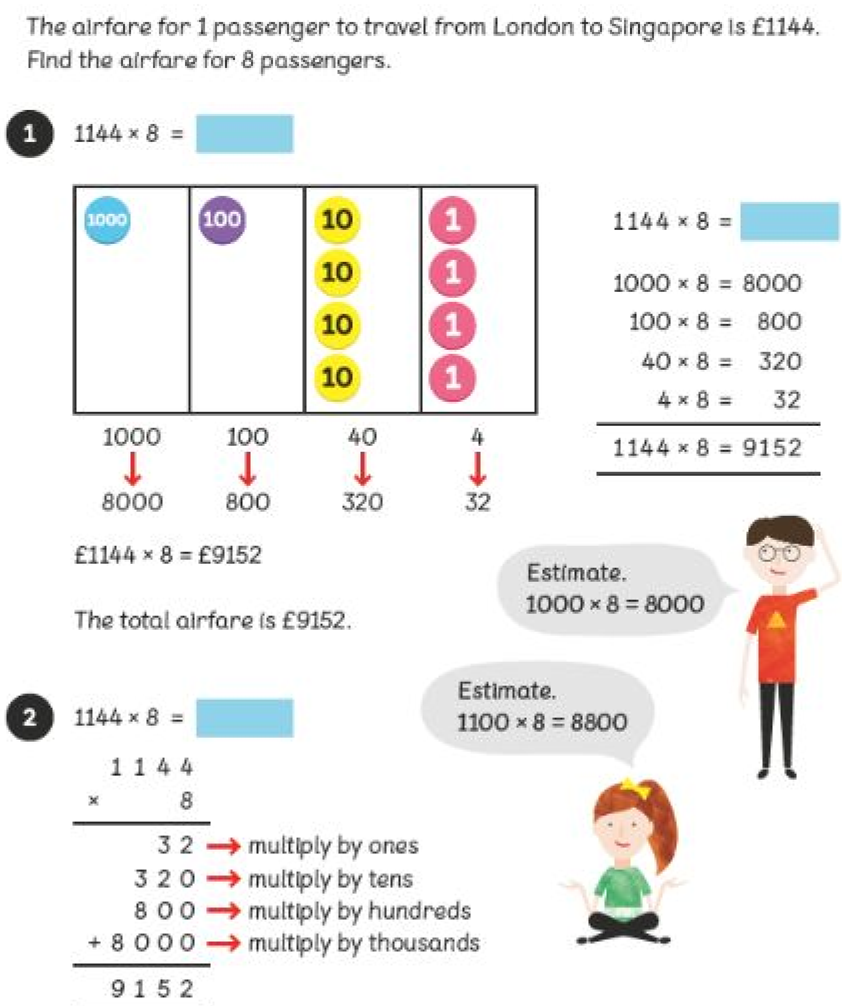
Note: the short multiplication method is deemed to be the most efficient written method for multiplying more complex numbers, and as such, it is hoped that most children will become proficient in using this method; however, it is acceptable for pupils to use the number bond or expanded method, whatever method feels most comfortable for them.

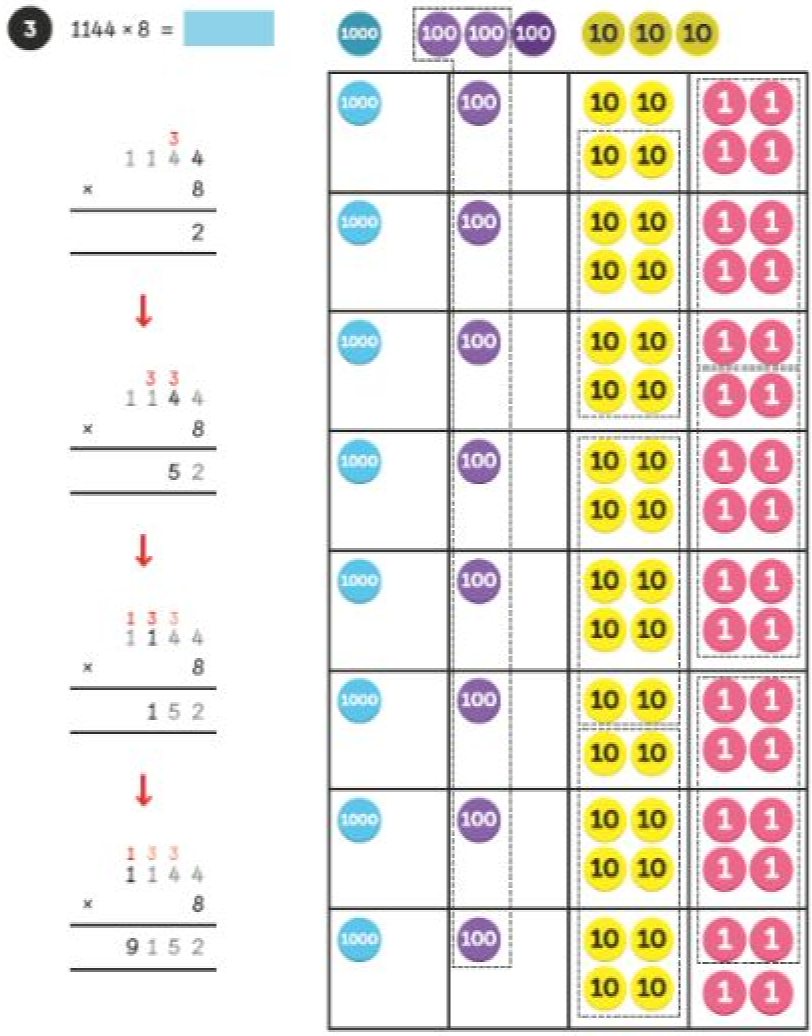
Multiplication – Year 5:

As pupils progress into Year 5, they will continue to build on their previous knowledge of multiplication, and their developing understanding of numbers of greater value. At this stage, pupils are multiplying numbers up to 4-digits by single-digit numbers, and 2-digit by 2-digit numbers (long multiplication).

Multiplying a 4-digit number by a single-digit number:

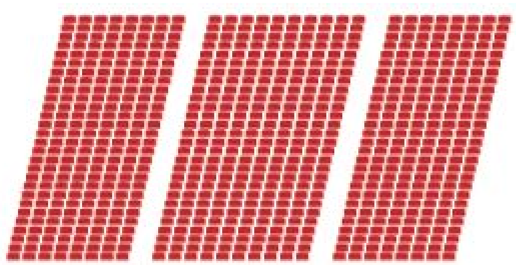
Pupils should apply their knowledge of multiplying 3-digit numbers to now calculating with larger numbers. Pupils should be moving towards using the most efficient method of calculation, short multiplication, but it is acceptable to use any other accurate written method





Multiplying a 2-digit number by a 2-digit number:

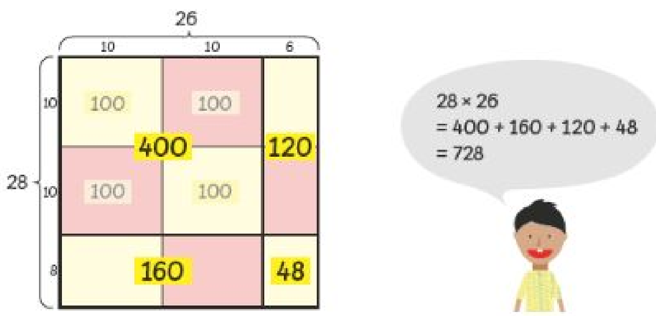
Pupils now progress to multiplying 2-digit by 2-digit numbers. At this stage, pupils should use whatever method they feel more comfortable with, with the long term view of becoming confident in long multiplication, the most efficient method. All methods shown will be modelled for pupils by teachers, as shown:



How many seats are there in this theatre?

Grid method:

There are 28 rows.
Each row consists of 26 seats.

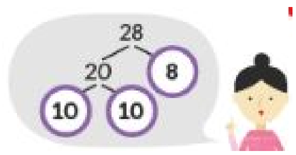


There are 728 seats.

Number Bond method:

There are 28 rows.
Each row consists of 26 seats.

28 × 26 = ▭



10 × 26 = 260
10 × 26 = 260
8 × 26 = 208

28 × 26 = 728

There are 728 seats.

$$\begin{array}{r} {\scriptstyle 4} \\ 2\,6 \\ \times \quad 8 \\ \hline 2\,0\,8 \\ \hline \end{array}$$

Column method (long multiplication):

28 × 26 = ▭

$$\begin{array}{r} {\scriptstyle 4} \\ 2\,8 \\ \times\ 2\,6 \\ \hline 8 \end{array} \longrightarrow \begin{array}{r} {\scriptstyle 4} \\ 2\,8 \\ \times\ 2\,6 \\ \hline 1\,6\,8 \end{array}$$

$$\begin{array}{r} {\scriptstyle 1} \\ 2\,8 \\ \times\ 2\,6 \\ \hline 1\,6\,8 \\ 6 \end{array} \longrightarrow \begin{array}{r} {\scriptstyle 1} \\ 2\,8 \\ \times\ 2\,6 \\ \hline 1\,6\,8 \\ 5\,6 \end{array}$$

$$\begin{array}{r} {\scriptstyle 1} \\ {\scriptstyle 4} \\ 2\,8 \\ \times \quad 2\,6 \\ \hline 1\,6\,8 \\ +\ 5\,6 \\ \hline 7\,2\,8 \\ \hline \end{array} \quad \begin{array}{l} \\ \\ \\ \\ \longrightarrow 28 \times 6 \\ \longrightarrow 28 \times 20 \\ \\ \end{array}$$

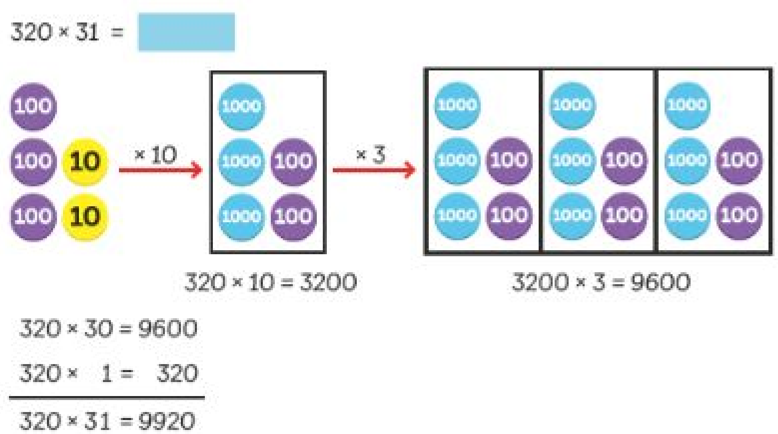
Multiplication – Year 6:

In year 6, pupils continue to build upon their previous learning of multiplying numbers of increasing complexity (4-digit by 2-digit numbers), using the column method of long multiplication.

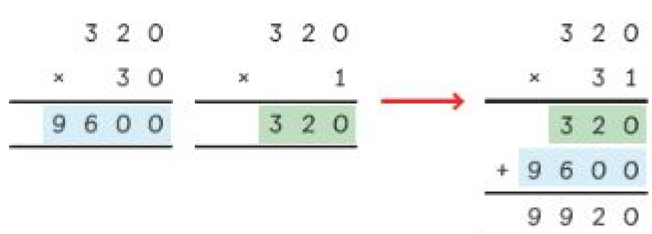
Multiplying 3 and 4-digit by 2-digit numbers:

Some pupils may still be more comfortable using the number bond method, while most should be using the long multiplication method.

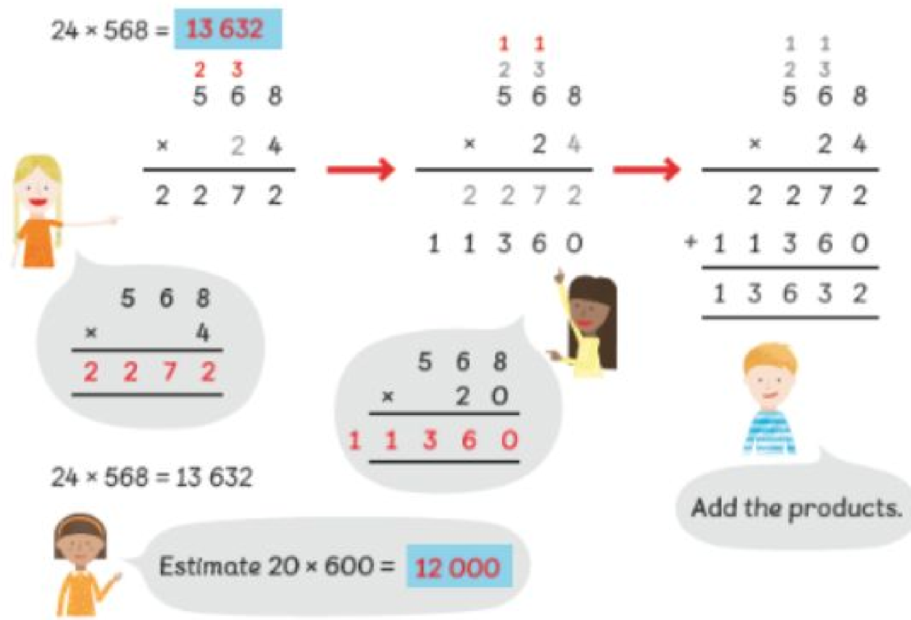
Number bond method:

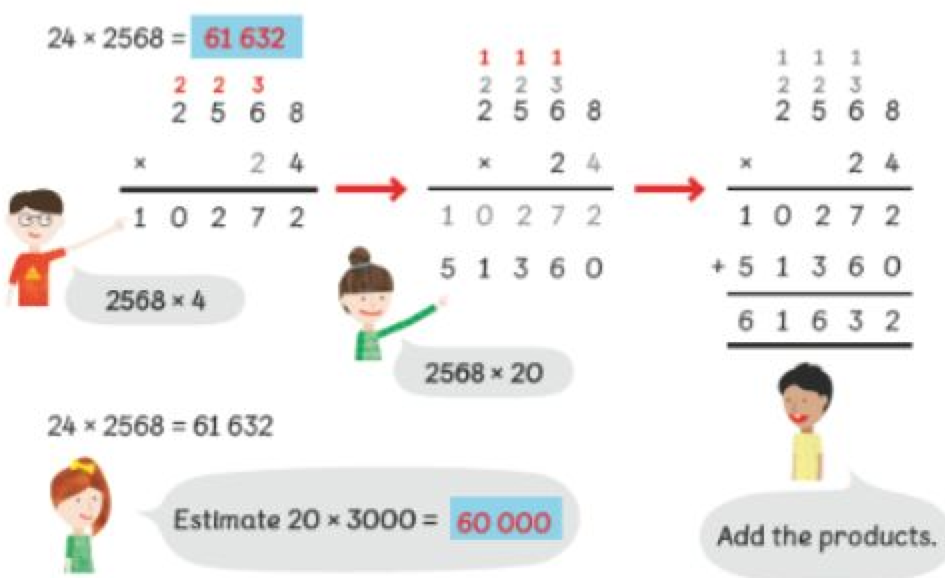


Long multiplication method:



Pupils should become more proficient in this method with practice, moving onto renaming and more complex numbers as they progress. Estimation should be used to help check how accurate answers are.





4. Stages in Division

Division – Early Years

Children will engage in a wide variety of songs and rhymes, games and activities.

In practical activities and through discussion, they will begin to solve problems involving halving and sharing.

Share the apples between two people.

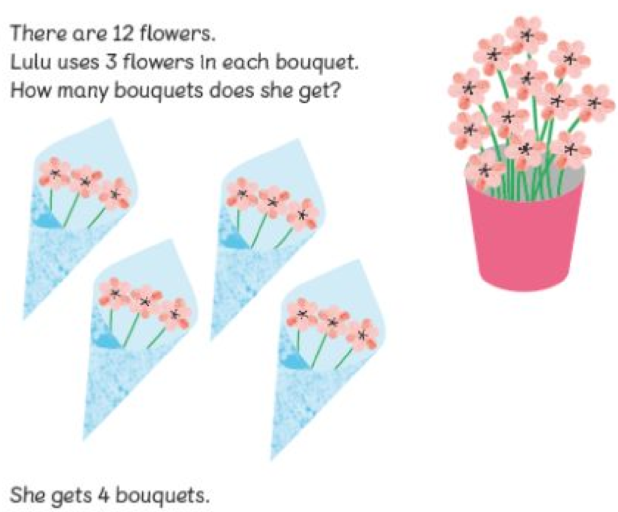
'Half of the apples for you and half for me. We have three apples each'

Division – Year 1:

Children will begin understanding the concept of division by distinguishing the difference between grouping and sharing in order to find out how many groups of the same number make up the whole.

Grouping:

When grouping, children will determine how to divide even numbers into equal groups using concrete materials.



In this example, the 12 flowers have been equally shared to make 4 **groups**. Using concrete objects to represent the flowers and paper plates as the 'groups' allow pupils to demonstrate their understanding in a practical way.

Sharing:

When sharing, pupils need to be able to determine how many objects will be included in each group in order to share equally.

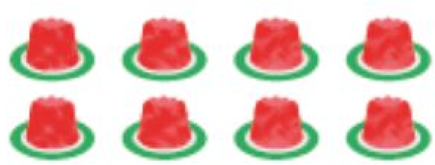
Division – Year 2:

Pupils will continue learning that grouping and sharing are ways of dividing and use the division (÷) and equals (=) signs, having previously had the opportunity to understand what multiplication means and what it looks like. They also investigate the links between multiplication and division.

Grouping:

Pupils now link their understanding of grouping to writing a division equation.

Circle to show groups of 2. How many groups are there?



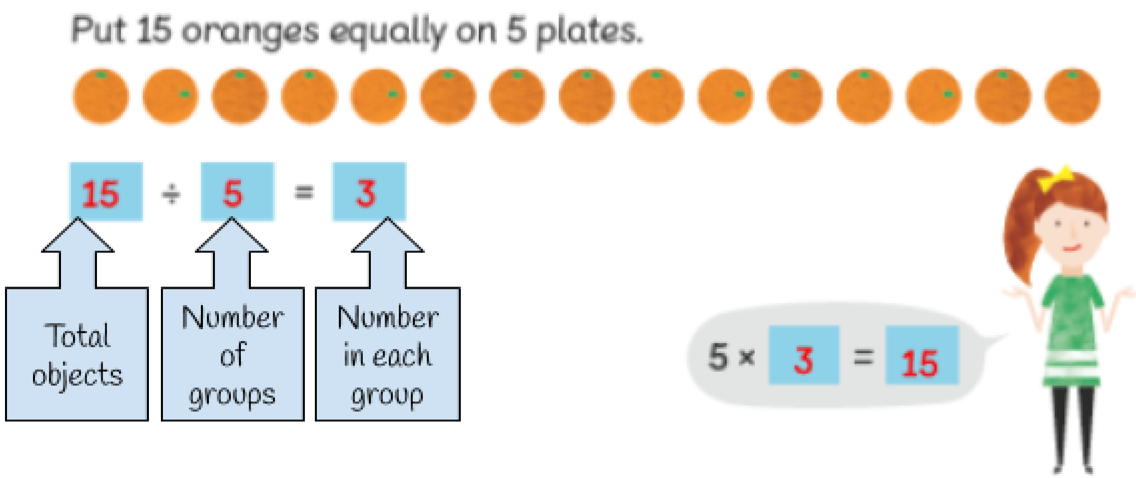
$\boxed{8} \div \boxed{2} = \boxed{4}$

Total objects	Number in each group	Number of groups

There are $\boxed{4}$ groups.

Sharing:

Pupils now link their understanding of sharing to writing a division equation, and then using known multiplication facts to check answers.

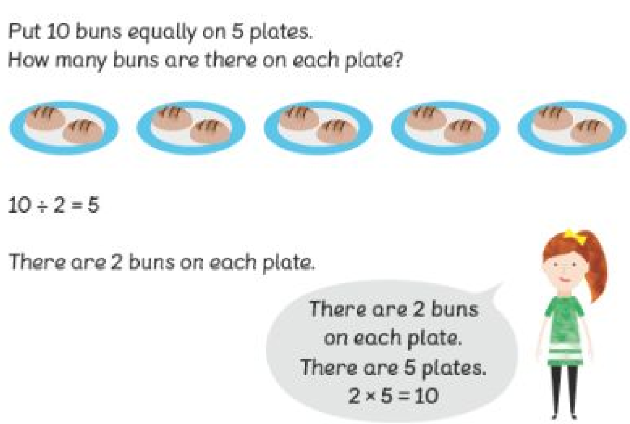


Dividing by 2, 5 and 10:

Children will then spend time using their knowledge of grouping and sharing and apply it to problems involving the 2, 5 and 10 times tables. There will be at least one lesson focussing on each times table.

Multiplication and Division:

Children will continue to make links between multiplication and division and use this knowledge to make a 'family' of multiplication and division facts.



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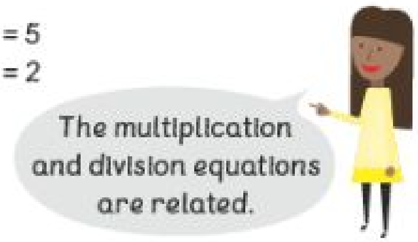
We can make a family of multiplication and division facts.

$5 \times 2 = 10$ ———— $10 \div 2 = 5$

$2 \times 5 = 10$ ———— $10 \div 5 = 2$



Solving word problems:

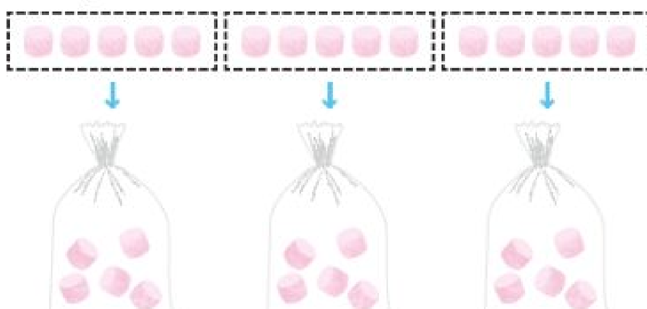
In order for pupils to apply their knowledge of multiplication and division, they will now have to solve word problems involving division within the division facts of the 2, 5 and 10 times tables. Pupils should first solve problems using concrete resources, moving onto pictorial representations and then abstract representations (number only) once confident.

Ruby has 15 marshmallows.
She packs 5 marshmallows into each bag.
How many bags does Ruby need?

Method 1 Use counters to stand for marshmallows.

Use a cup for each bag.

Method 2 Draw a picture.



Method 3 Use a division equation.

$15 \div 5 = 3$

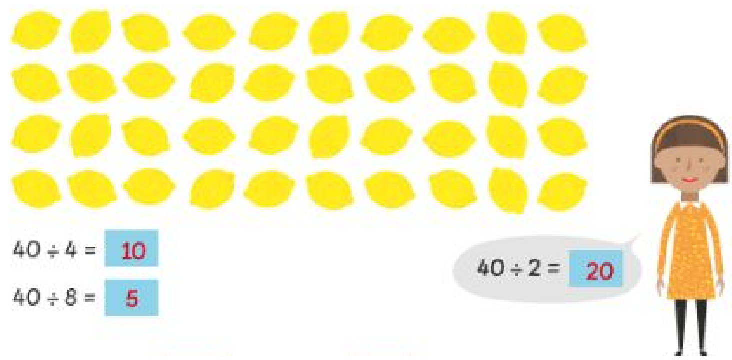
Ruby needs 3 bags.

Division – Year 3:

Dividing by 3, 4 & 8:

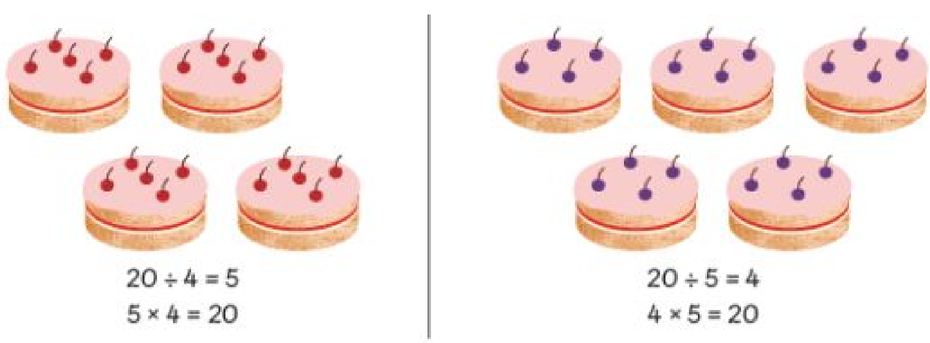
Before learning about division in year 3, pupils will have spent time becoming proficient in the multiplication tables for 3, 4 & 8. They should then be able to use this knowledge of times tables to be able to divide by 3, 4 & 8. They continue to learn about division in the context of how it can be seen in two different ways: grouping and sharing. Again, as with learning multiplication facts, pupils are encouraged to see the relationships between numbers, such as 2, 4 & 8, leading pupils to see that when dividing 8, the answer will be half of what it is when divided by 4, and so on.

How many lemons would there be in each group if we put them into 4 equal groups? 8 equal groups? 2 equal groups?



Multiplying and dividing:

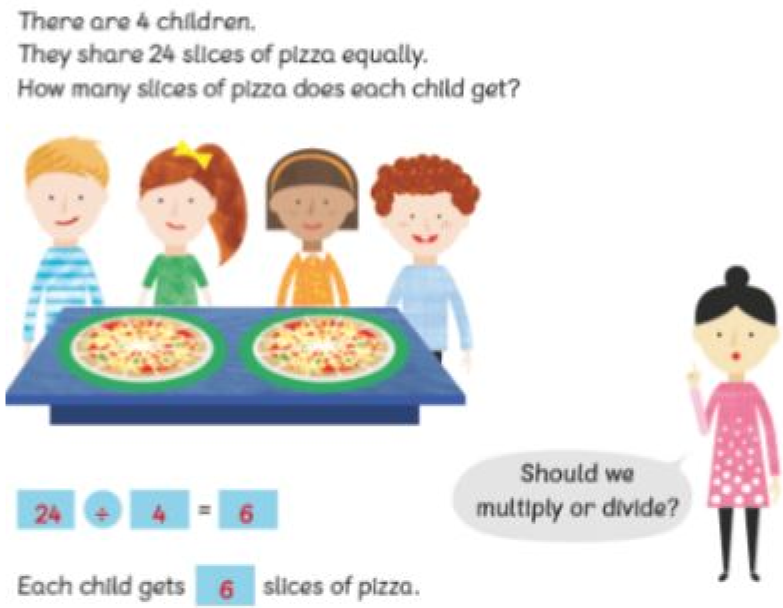
Pupils continue to see the relationship between division and multiplication, and write a 'family' of multiplication and division facts to show this.



We can make a family of multiplication and division equations.

Word problems:

Once pupils have covered the multiplication and division of 3, 4 and 8, they will then get to use their experience of multiplication and division to solve word problems, deciding which operation is necessary to find the solution.

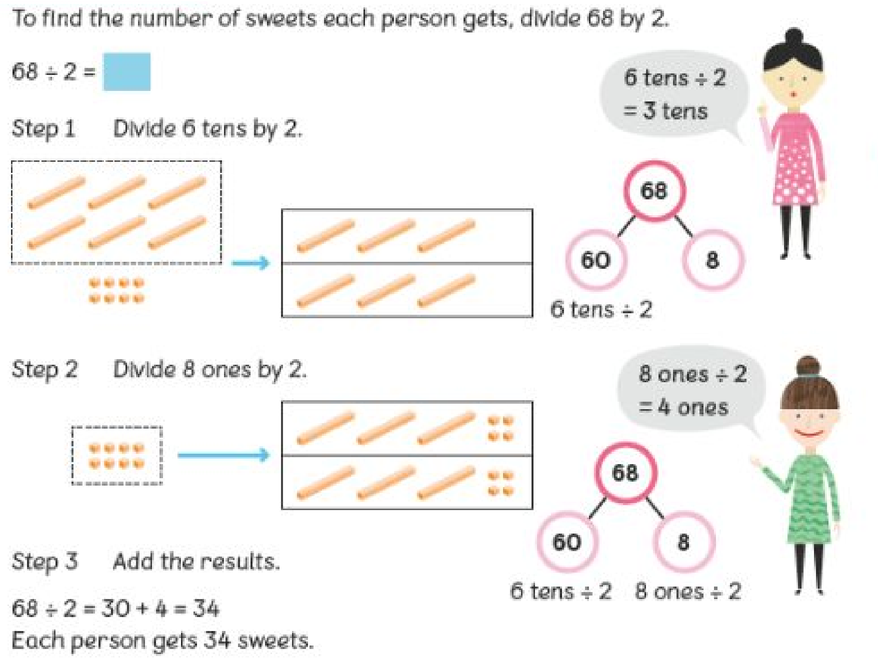


Dividing 2-digit numbers:

Having been taught how to multiply 2-digit numbers, pupils will be taught how to divide them, first with simple division, and then division with regrouping. Decomposing numbers (partitioning/breaking apart) is critical in making both multiplication and division manageable for pupils.

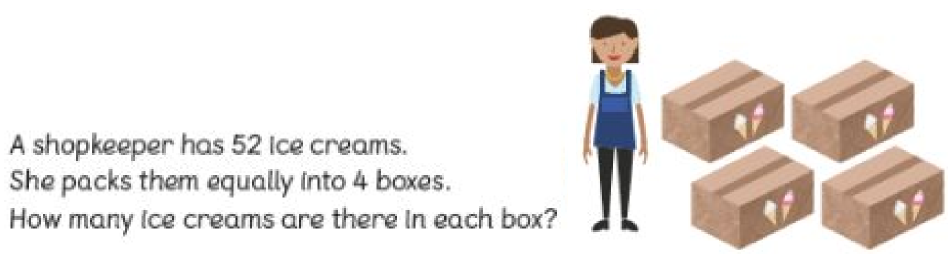
1. Simple dividing:

Using Base 10 materials, pupils will explore how to solve the problem.



2. Dividing with regrouping:

Next, pupils will solve division problems where tens are regrouped as ones, in order to be shared equally.

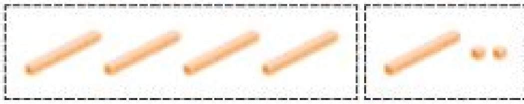


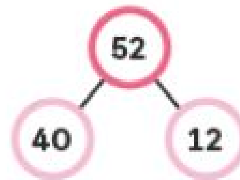
In the problem above, children will see that 5 tens and 2 ones cannot be shared equally into 4 groups, (having sharing 4 tens equally, they will have 1 ten and 2 ones left over) therefore they will need to decompose 52 into 'friendlier' numbers.

To find the number of ice creams in each box, divide 52 by 4.

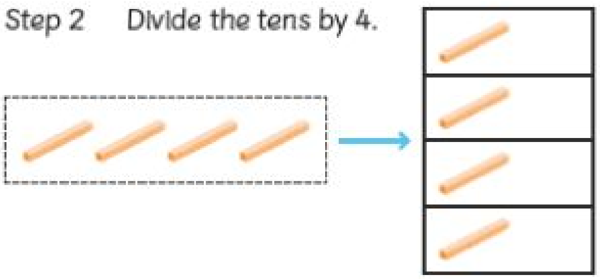
$52 \div 4 = \boxed{}$

Step 1 Split 52 into 40 and 12.

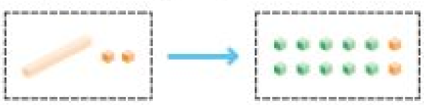




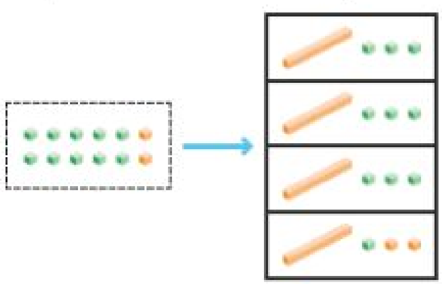
Step 2 Divide the tens by 4.



Step 3 Regroup 1 ten into 10 ones.



Step 4 Divide the ones by 4.



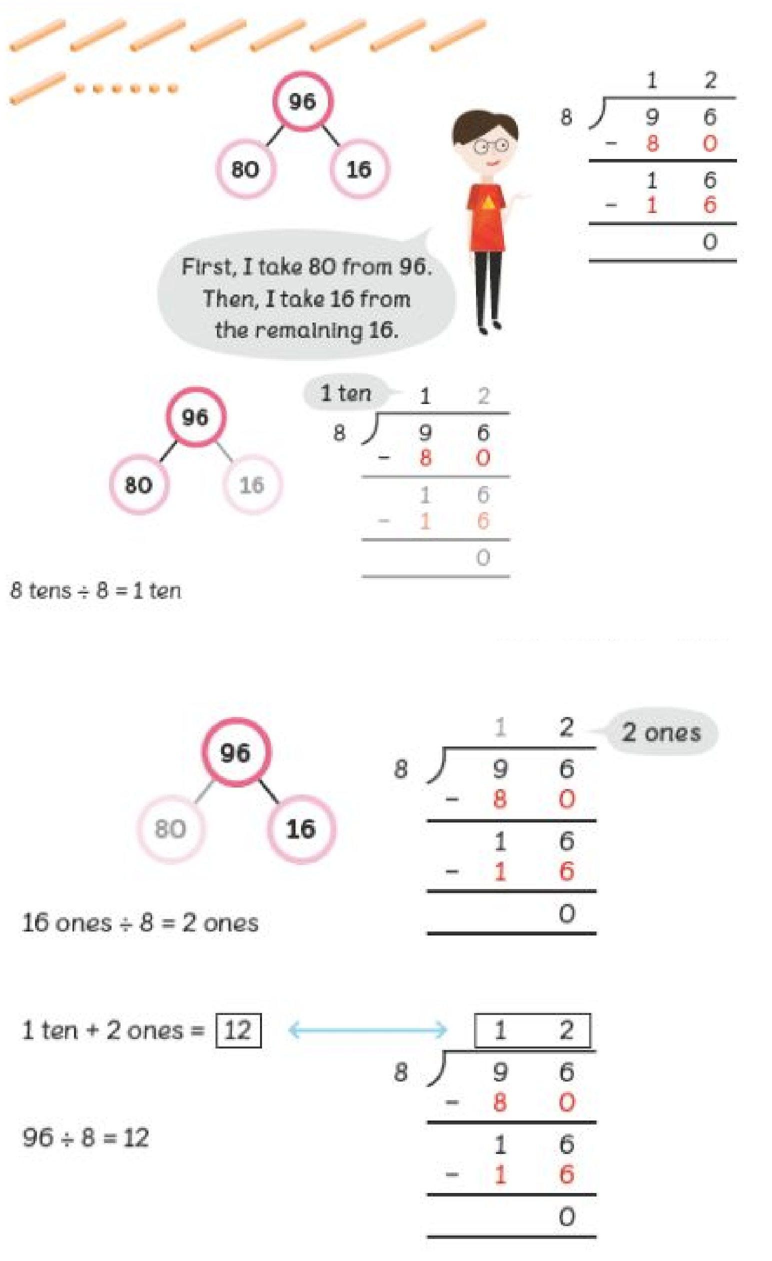
Step 5 Add the results.

$52 \div 4 = 10 + 3 = 13$

There are 13 ice creams in each box.

3. Long division:

Finally, pupils are shown how to complete a division problem using the standard algorithm (long division). Pupils will be taught this method alongside the number bond method, as in this example: 98 divided by 8:



Please note: at this stage of division, pupils should use whichever method they feel most confident in. There is no requirement to use the long division method.

Division – Year 4:

Following Year 3, pupils will now progress to learning to divide by 6, 7, 9, 11 & 12, relating their learning to knowledge of these multiplication tables. They will be taught how to use number patterns to help solve calculations, such as with the 3, 6 & 9 tables. They will learn vocabulary such as *inverse, dividend, divisor* and *quotient* in relation to division, as well as continuing to explore the difference in grouping and sharing as they relate to division.

A roller coaster with 6 carriages, all of the same size, can seat 24 people.
How many people does each carriage seat?

In this example, the dividend (total number to be shared) is 24, the divisor (number of groups) is 6 and the quotient is 4.

Pupils will encounter dividing with remainders for the first time, and well as continue exploring using the standard long division algorithm.

Dividing with remainders:

Before learning about remainders, pupils should be confident in the fact that groups need to be the same to divide equally.

Ruby tries to put 39 apples equally into 4 baskets.

39 ÷ 4 = 9 remainder 3

There are 9 apples in each basket.

3 apples are left over.

Dividing 2-digit numbers:

Pupils learn to divide 2-digit numbers using concrete resources, the number bond and long multiplication methods. Showing each methods side-by-side helps to reinforce their conceptual understanding of the process of division.

Mr. Smith has a collection of 68 old postcards. Ruby and Ravi share them equally. How many postcards should each take?

$68 \div 2 = \square$

10	10	10	10	10	10	1	1	1	1	1	1	1	1

Method 1

$$\begin{array}{ccc} & 68 & \\ 60 & & 8 \end{array}$$

68 = 60 + 8

$$\begin{array}{ccc} & 68 & \\ 60 & & 8 \\ 30 & & 4 \end{array}$$

Divide 60. Divide 8.

Method 2

$$\begin{array}{r} 2 \overline{)\,6\;\;8} \\ -\,6\;\;\;\; \\ \hline 8 \\ -\;\;8 \\ \hline 0 \\ \hline \end{array}$$

6 tens ÷ 2

$$\begin{array}{r} 3\;\;\;\; \\ 2 \overline{)\,6\;\;8} \\ -\,6\;\;\;\; \\ \hline 8 \\ -\;\;8 \\ \hline 0 \\ \hline \end{array}$$

8 ones ÷ 2

$$\begin{array}{r} 3\;\;4 \\ 2 \overline{)\,6\;\;8} \\ -\,6\;\;\;\; \\ \hline 8 \\ -\;\;8 \\ \hline 0 \\ \hline \end{array}$$

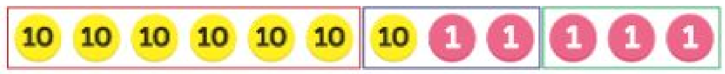
Each should take 34 postcards.

$68 \div 2 = 34$

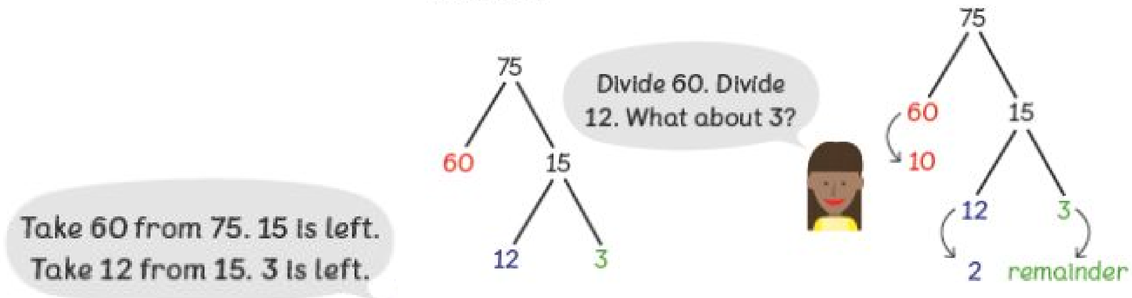
Pupils will now combine their knowledge of dividing 2-digit numbers using written methods and remainders.

Is it possible to put 75 children into 6 equal groups?

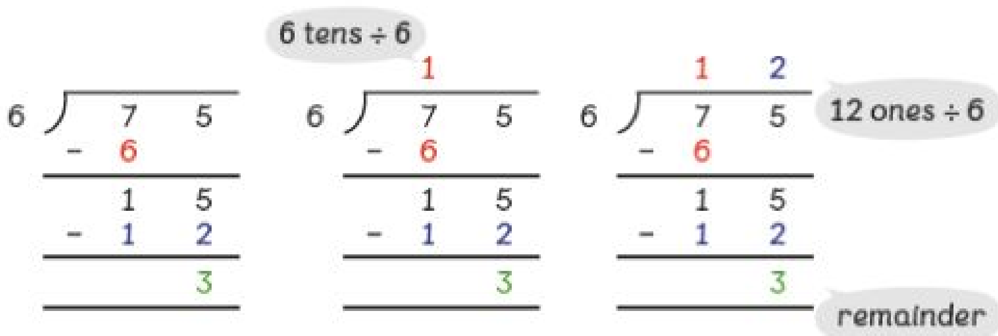
$75 \div 6 = \boxed{}$



Method 1



Method 2

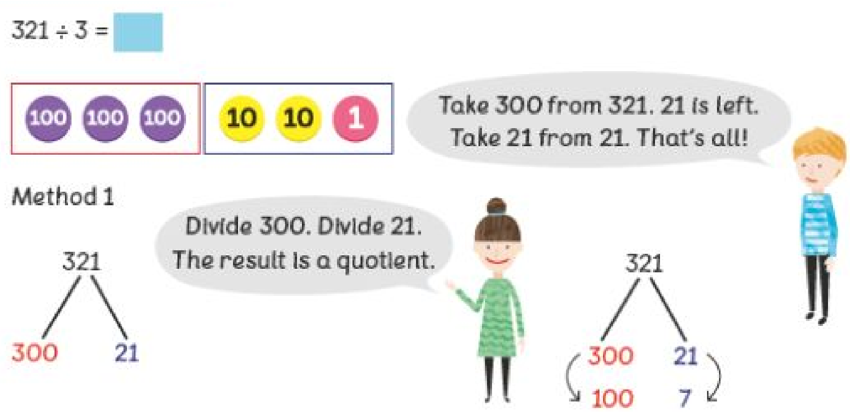


$75 \div 6 = 12$ remainder 3

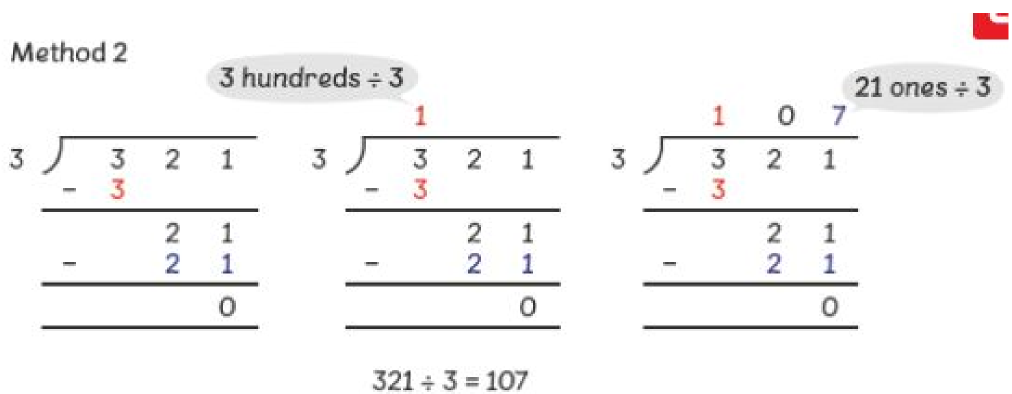
quotient

It is not possible to put 75 children into 6 equal groups.

Dividing a 3-digit number with renaming:



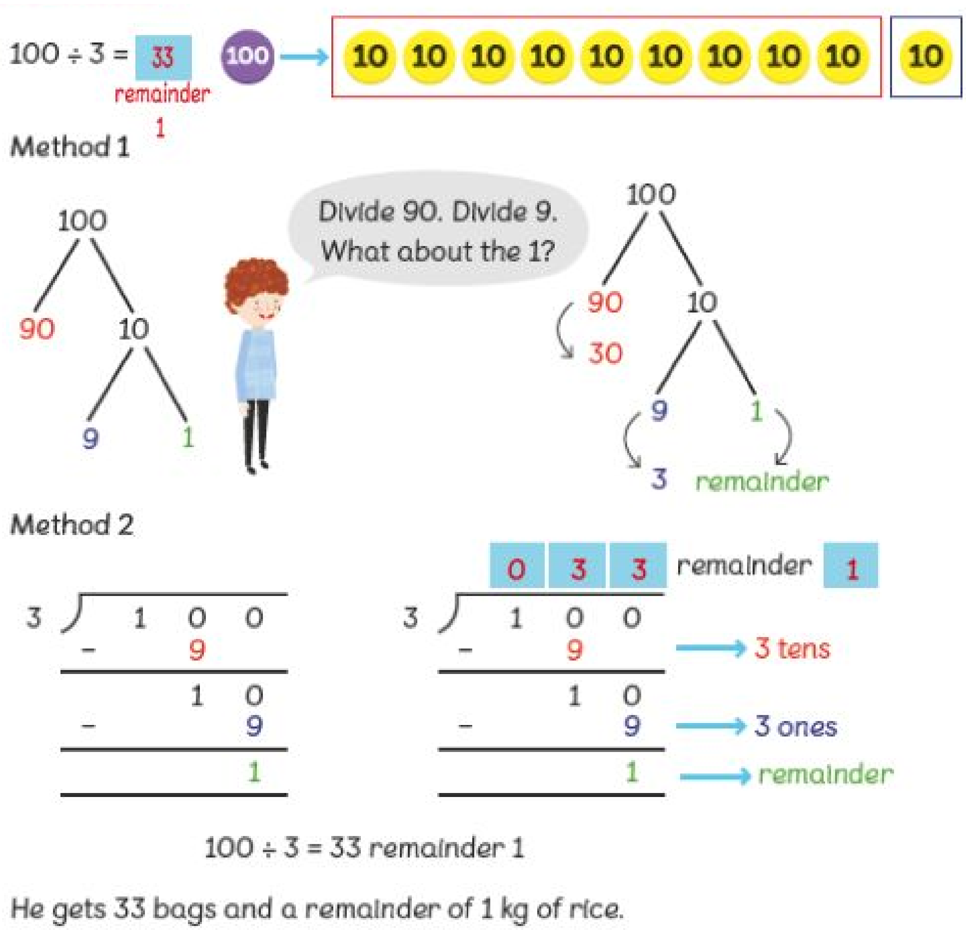
In this example, 2 tens cannot be shared into 3 groups, so they are renamed as 20 ones. The method below shows the same equation solved using long division.



Dividing 3-digit numbers with remainders:

Finally, pupils will experience dividing 3-digit numbers where there will be a remainder. Again, pupils will be exposed to a number of methods, and should use whichever method they are most confident in. there is no statutory requirement to use formal written methods at this stage.

A shopkeeper repacks 100 kg of rice into 3-kg bags to sell.
How many bags does he get?



Division- Year 5:

In Year 5, pupils learn to divide, with remainders, using multiple methods, including number bonds and long and division. This is the first time that pupils are required to use a formal written division method.

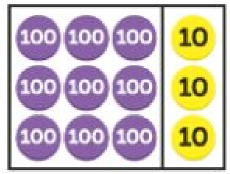
Dividing 3-digit numbers:

Building on their Year 4 learning, pupils will be shown how the number bond method and long division are asking the same questions. They will be encouraged to use the long division method as a more efficient method.

Think about how 3 people can share £930 equally.

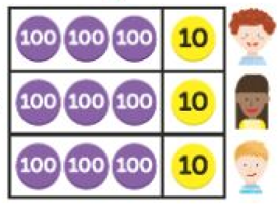
1 930 ÷ 3 = ☐

$$\begin{array}{ccc} & 930 & \\ / & & \backslash \\ 900 & & 30 \end{array}$$

930


30 ÷ 3 = 10

900 ÷ 3 = 300

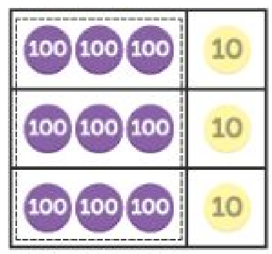
930 ÷ 3


930 ÷ 3 = 310

2 930 ÷ 3 = 310

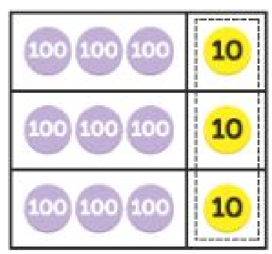
$$\begin{array}{ccc} & 930 & \\ / & & \backslash \\ 900 & & \boxed{30} \end{array}$$

$$\begin{array}{r|rrr} 3 & 9 & 3 & 0 \\ - & 9 & 0 & 0 \\ \hline \end{array}$$



$$\begin{array}{ccc} & 930 & \\ / & & \backslash \\ 900 & & 30 \end{array}$$

$$\begin{array}{r|rrr} 3 & 9 & 3 & 0 \\ - & 9 & 0 & 0 \\ \hline & & 3 & 0 \\ - & & 3 & 0 \\ \hline & & & 0 \\ \hline \end{array}$$

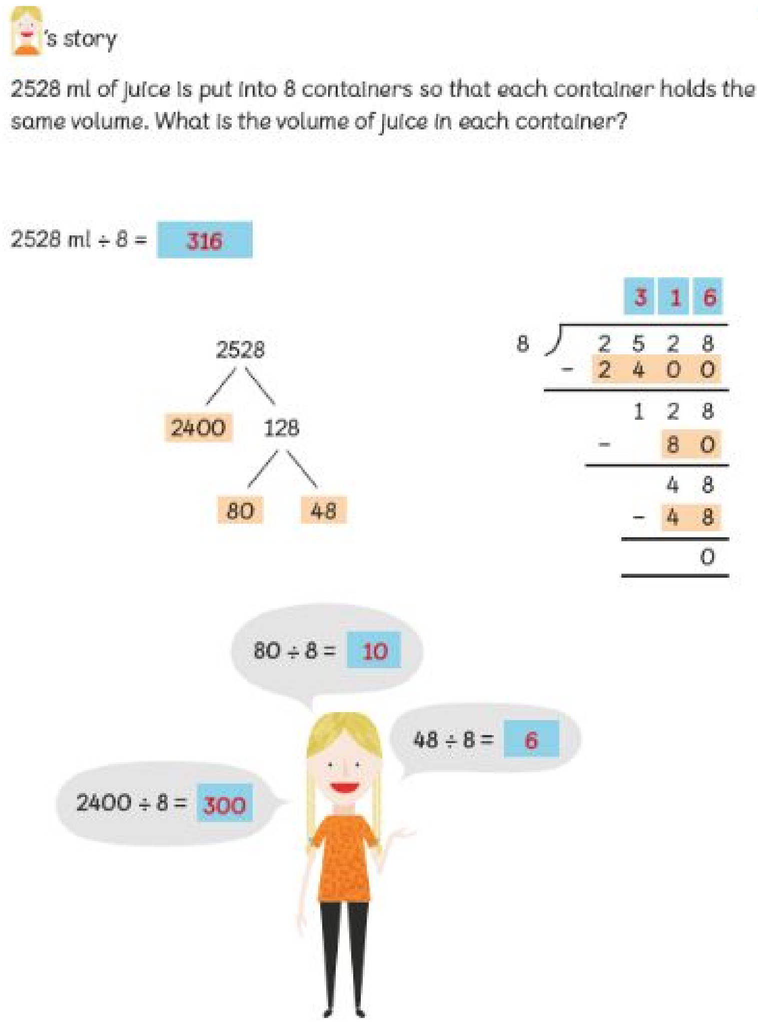


$$\begin{array}{ccc} & 930 & \\ / & & \backslash \\ 900 & & 30 \\ \downarrow & & \downarrow \\ 300 & & 10 \end{array}$$

$$\begin{array}{r|rrr} & 3 & 1 & 0 \\ \hline 3 & 9 & 3 & 0 \\ - & 9 & 0 & 0 & \longrightarrow 300 \\ \hline & & 3 & 0 \\ - & & 3 & 0 & \longrightarrow 10 \\ \hline & & & 0 \\ \hline \end{array}$$

Dividing 4-digit numbers:

By continuing to use the number bond and long division method side by side, pupils should be gaining a better conceptual understanding of division, rather than rote learning of the procedure with the understanding that underpins it.



Dividing 3 & 4-digit numbers with remainders (incl. short division):

Once pupils have gained a good conceptual understanding of the number bond and long division methods, they will be exposed to two different short division methods. With support, they will be shown how there are many representations of solving division problems, all underpinned by the same place value understanding, and they should decide when is best to use each method.

 asked his mother for help to find the value of 376 ÷ 5 in solving a word problem about putting 376 children in 5 equal groups.

's mother's method

$$\begin{array}{r} 7\ 5 \text{ remainder } 1 \\ 5\,\big)\overline{3\ 7\ {}^{2}6} \end{array}$$

His father chipped in.

's father's method

$$\begin{array}{l} 5\,\big\backslash\underline{3\ 7\ {}^{2}6} \\ \quad\ \ 7\ 5 \text{ remainder } 1 \end{array}$$

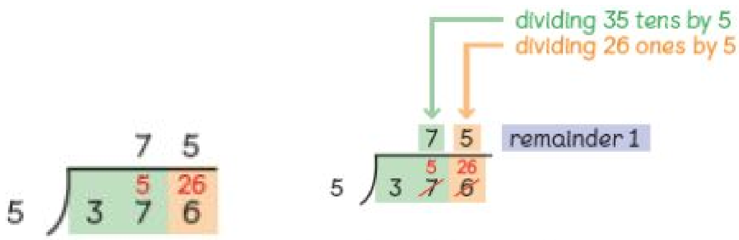
This is what  learnt in school.

's method

$$\begin{array}{rr} & \boxed{7}\,\boxed{5} \\ 5 & \big)\overline{3\ 7\ 6} \\ & -\ 3\ 5\ 0 \\ \hline & 2\ 6 \\ & -\ 2\ 5 \\ \hline & 1 \\ \hline \end{array}$$

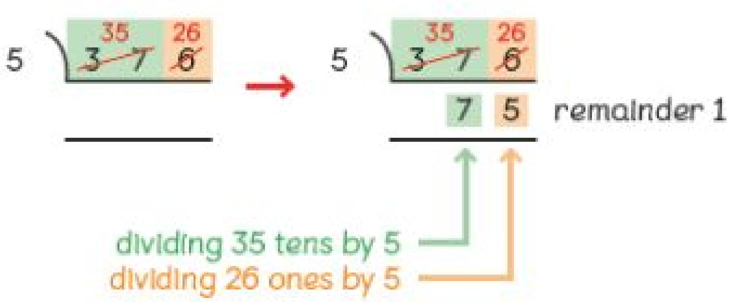
Work out the methods used by 's mother and father.

My mother's method:



376 ÷ 5 = 75 remainder 1

My father's method:



Division – Year 6:

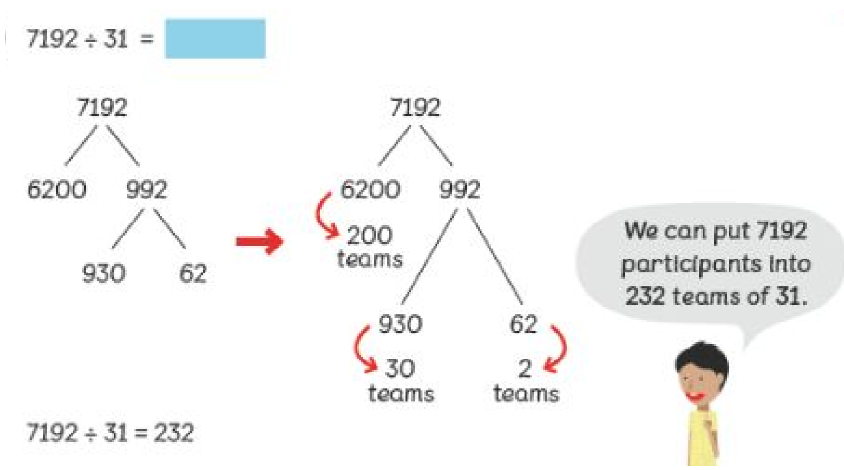
In Year 6, pupils will begin dividing numbers up to 4-digits by 2-digit numbers. They will be using multiples of the divisor to support decomposing the dividend, as shown in the examples below, and be exposed to number bond, long and short division strategies. Formal written methods must be used in Year 6.

Dividing by 2-digit numbers:

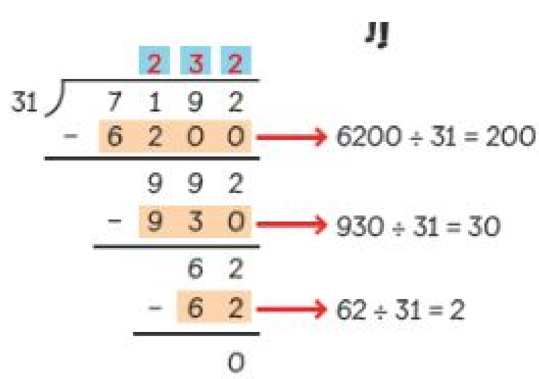
Example 1: 7,192 people registered for a sports camps. Can we put them into groups of 31?

Pupils will need to decompose 7,192 into multiples of 31 (31, 62, 93) and use their knowledge of related facts, as shown:

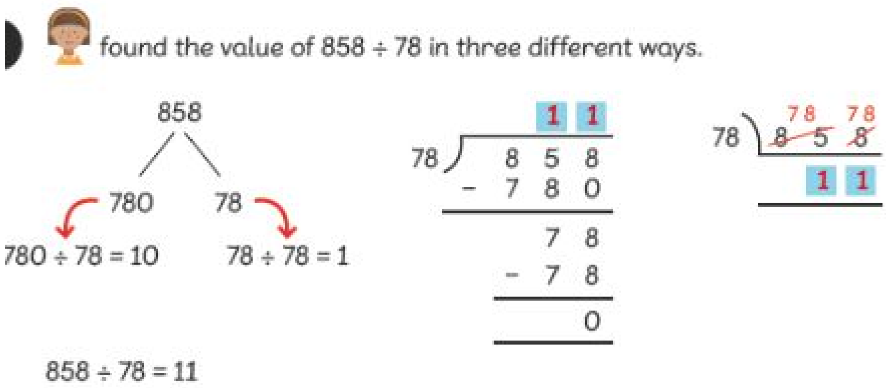
Number bond method:



Long division:

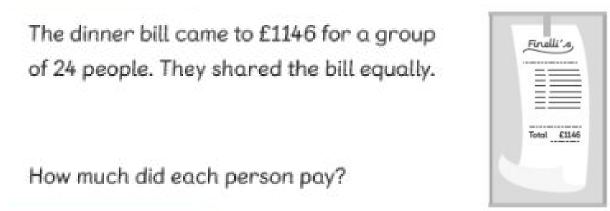


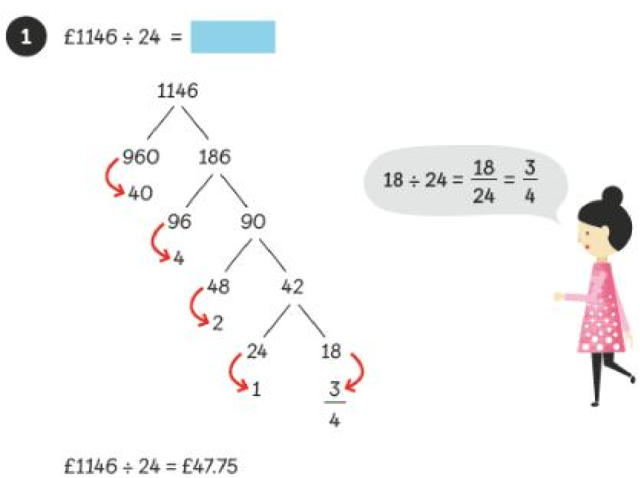
Example 2: For all three methods, pupils need to decompose 858 into numbers that are multiples of 78.



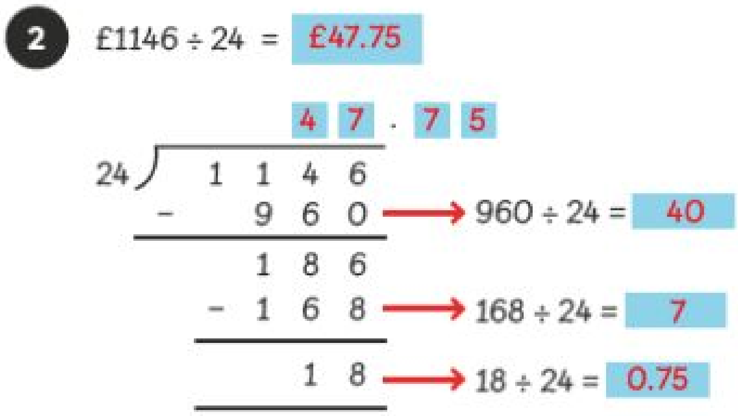
Dividing by 2-digit numbers with remainders:

In Year 6, pupils will be asked to represent remainders in context, for example if they were dividing an amount of money, as shown below:





What should we do with the remainder 18? When reduced to the simplest form 18/24 is 3/4. In this context, rather than say 3/4 of a pound, we convert to a decimal – £0.75



Alternatively, pupils could use factors of 24 to find the answer, rather than dividing by a 2-digit number; the use a bar model can support the understanding of the factors involved (2x4x6 =24):